PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Determinations of the Fundamental Standards of Length in Terms of Wave-Lengths of Light

J. E. Sears, H. Barrell and W. H. Johnson

Phil. Trans. R. Soc. Lond. A 1934 **233**, 143-216
doi: 10.1098/rsta.1934.0016

III. *Determinations of the Fundamental Standards of Length in Terms of Wave-Lengths of Light.*

By J. E. Sears, *Junr., C.B.E., M.A., M.I.Mech.E., and* H. Barrell, *B.Sc., A.R.C.Sc., D.I.C.*, Metrology Department, The National Physical Laboratory.

(*Communicated by Sir* Joseph Petavel, *F.R.S.*)

(Received December 1, 1933—Read March 1, 1934.)

Contents.

VOL. CCXXXIII—A 723. (Price 9*s.*) U Published June 22, 1934.

1. *Introduction.*

A new apparatus for determining the relationship between wave-lengths of light and the fundamental standards of length has been previously described.* Definitive determinations have now been completed of the lengths of the yard and metre in terms of the wave-length of the cadmium red radiation, both in air and in vacuum, and the present paper gives the results of these determinations.

Previous determinations have been made by MICHELSON and BENOÎT,† by BENOÎT, FABRY, and PEROT,‡ and by WATANABE and IMAIZUMI,§ of the length of the metre in terms of the cadmium red radiation in air, and these results, after adjustment as nearly as possible from the experimental data available to uniform conditions, agree with each other and with that obtained in the present work, within a total range of four parts in ten millions, a range which is not greater than may reasonably be attributed to the experimental errors of determination of the lengths of the different copies of the metre against which the several comparisons have been made.

No previous direct measurement has been made of the length of either the yard or the metre in terms of wave-lengths in vacuum. The paper records the first independent determination of these important relationships, and incidentally affords a new direct determination of the refractive index of dry air, free of carbon dioxide, which is in good agreement with that given by PÉRARD,|| but differs appreciably from that given by MEGGERS and PETERS.¶

A further incidental result is a new determination of the ratio of the yard to the metre which is in good agreement with the most recent determination of this ratio by purely metrological methods.**

The paper concludes with some suggestions as to the possibility of eventually defining the units of length in terms of the wave-length of some suitably chosen radiation of visible light instead of, as at present, by reference to material standard bars. In this connection an important development has taken place recently, for at the meeting of the International Committee of Weights and Measures held in October, 1933,†† a special sub-committee was appointed, with the sanction of the Eighth General Conference on Weights and Measures, to study the principles involved in the adoption of such a definition.

Before proceeding to the details of the present work it will be useful to recall briefly

---

* SEARS and BARRELL, 'Phil. Trans.,' A, vol. 231, p. 75 (1932).

† 'Trav. Bur. int. Pds. Mes.,' vol. 11, p. 85 (1895).

‡ 'Trav. Bur. int. Pds. Mes.,' vol. 15, p. 131 (1913).

§ 'Proc. Imp. Acad. Tokyo,' vol. 4, p. 351 (1928).

|| 'Trav. Bur. int. Pds. Mes.,' vol. 19, p. 78 (1932).

¶ 'Bull. Bur. Stand.,' vol. 14, p. 697 (1918–1919).

** SEARS, JOHNSON, and JOLLY, 'Phil. Trans.,' A, vol. 227, p. 298 (1928).

†† 'Proc.-verb. Com. int. Poids Mes.' (1933), in the press.

the main features of the method and apparatus employed. The basic principle of the method is similar to that used by BENOÎT, FABRY, and PEROT, *loc. cit.*, but with important modifications in experimental detail. It consists in determining in turn the number of wave-lengths contained in the distances between the semi-reflecting surfaces of three FABRY-PEROT étalons whose lengths are in the ratios $\frac{1}{12}$ or $\frac{1}{9} : \frac{1}{3} : 1$. The étalons in the present apparatus are of tubular form, made of invar, with flat parallel chromium-plated ends to which the flat semi-silvered glass or quartz plates are directly contacted, thus forming air-tight joints which allow the étalons to be either evacuated or filled with air under controlled and known conditions.

The longest étalon is just over 1 m. in length, and contains a steel gauge of X-shaped cross-section with optically polished flat parallel end-surfaces. It also has embodied in its construction a platinum-resistance thermometer by means of which its temperature, and that of the X-gauge, may be determined to an accuracy of $\pm 0{\cdot}001^\circ$ C.

Two X-gauges are provided, one a metre and the other a yard in length. The first stage in the work is the determination of the number of wave-lengths corresponding to the lengths of these gauges. The procedure is as follows: first the number of wave-lengths in the shortest étalon is determined, precisely as in the BENOÎT-FABRY-PEROT method, by measurement of circular fringes formed in transmitted monochromatic light. The length of the second (intermediate) étalon is then deduced by comparison with the first, using the method of optical multiplication employing BREWSTER's fringes in white light. The length of the longest étalon is similarly compared with that of the intermediate étalon. This operation is carried out for each of the four channels left between the arms of the X-gauge and the wall of the surrounding tube. Finally, the distances between the ends of the X-gauge and the end-plates of the étalon are measured by means of circular fringes in reflected monochromatic light in a manner similar to the measurement of the first étalon. By subtraction the length of the X-gauge in wave-lengths is ascertained.

The comparison of the X-gauges with the existing standards of the corresponding units of length is carried out by the aid of certain special composite gauges. These gauges, which have been described elsewhere,* consist of end-bars of circular section, respectively half an inch shorter than the yard and metre, together with two rectangular blocks, each half an inch in length, which can be wrung on to their ends. The depth of the blocks is equal to the radius of the bar, and at the centre of the $\frac{1}{2}$-inch face of each, parallel to the end surfaces, a fine line is ruled. When one of the blocks is wrung on to each end of the bar, so that its graduated surface lies in the median plane of the bar, the whole constitutes a line-standard nominally a yard or a metre in length, which can be directly compared under microscopes with the fundamental standard or one of its copies. When one block at a time is wrung on to the centre of one end of the bar the whole constitutes an end-standard, again nominally one yard or one metre in length,

---

* 'Ann. Rep. Nat. Phys. Lab. Lond.,' p. 90 (1919).

which can be directly compared, either optically or in an end-measuring machine, with the X-gauge or any other end-standard.

In the present work the comparisons between the X-gauges and the composite bars were made optically by placing each in turn between a pair of fixed parallel semi-silvered glass plates, and measuring the gaps between the plates and the ends of the bars by means of the circular fringes formed in reflected monochromatic light. The comparisons of the composite gauges with the existing primary standards, or their copies, were made in the usual manner by observing them under microscopes in the comparator. By taking, in each stage of the work, the mean results of a whole group of comparisons in which the two blocks are subjected to a series of symmetrical interchanges on the end-faces of the bars, the actual lengths of the bars, and also of the end blocks, are entirely eliminated from the final determinations of the lengths of the X-gauges in terms of the existing standards.

A full account of the method and apparatus has been given in the previous paper, together with a description of the means employed, for example, for the control and measurement of temperature, and a discussion of the various corrections which have to be made to the observations. More detailed information regarding the experimental procedure is given in Section 2 and in Appendices II and III of this paper. Certain auxiliary apparatus has been provided since the previous paper was published. This consists chiefly of a small air-conditioning plant for removing moisture and carbon dioxide from the air admitted to the étalon system, and of a special barometer gauge for measuring the pressure of the air contained in the étalons. Descriptions of these two new items will be found in Appendix I.

2. *Experimental Notes.*

(*a*) *Source of Radiation used.*—Previous determinations of the length of the metre in terms of the wave-length of the cadmium red radiation have all been made in terms of the radiation produced from discharge tubes of the Michelson type. In the present work an Osram* lamp of the new hot-cathode discharge type has been used. The development of this type of lamp has resulted in a notable improvement in the efficiency of production of certain monochromatic radiations, and cadmium is one of the elements whose spectrum is easily produced in this way.

Among the advantages of using the Osram lamp in preference to the Michelson lamp are that it presents a large and brilliant source of highly monochromatic red radiation, and that it is simple to operate and has long life. It has been shown, by means of precise interferential comparisons,† that the wave-length of the red line of cadmium, whether produced from the Michelson lamp or from the Osram lamp, is the same to an

---

* Made by Studien-Gesellschaft für Elektrische-Beleuchtung m.b.H., Berlin.

† SEARS and BARRELL, 'Proc. Roy. Soc.,' A, vol. 139, p. 202 (1933).

accuracy of at least 1 part in 16,000,000, so that the use of the new lamp as a substitute for the more usual Michelson type is entirely justifiable.

(*b*) *Measurements of the X-gauges.*—In the measurements of the metre and yard X-gauges, which are subsequently referred to as X_M and X_Y respectively, 32 determinations were made on each bar, 16 being in air and 16 in vacuum. The determinations were made at the rate of two per day, one in air in the morning, and the other in vacuum in the afternoon. Upon the completion of the afternoon determination the étalons and reservoirs were filled with a fresh sample of conditioned air, which was left overnight to achieve the steady temperature of the thermostatically controlled enclosure in which the étalons and reservoirs were mounted.

In general only four days of each week, from Monday to Thursday, were occupied in observational work. Friday and, if necessary, Saturday morning were devoted to making the necessary changes and adjustments of the apparatus ready for the next week's work. The period between Saturday and Monday mornings gave ample opportunity for the apparatus to recover from the temperature disturbances caused by the opening of the enclosure and the manipulation of the apparatus.

The observations on X_M and X_Y therefore occupied a total period of eight weeks, and each week's observations were made under one condition of adjustment of the apparatus. Table I gives details of the eight conditions of the apparatus during the determinations of X_M and X_Y. Two étalons were used for the basic measurement of length in terms of light waves, namely, L_1 and L_2, their lengths being nominally equal to $L_6/12$ and $L_6/9$ respectively, where L_6 is the length of the longest étalon. L_1 and L_2 were suitably adjusted for parallelism and length at the beginning of the determinations and were not subsequently altered. The intermediate étalon L_5, of length nominally equal to $L_6/3$, and the longest étalon L_6 were used in all observations, but minor adjustments of parallelism and length were made as the different circumstances required.

TABLE I.—Arrangement of Apparatus for Determinations of X_M and X_Y.

Week.	Basic measurement.	X-Gauge.	Arrangement of X-Gauge in Étalon L_6.
1	L_1	X_M	Left end towards left end of L_6
2	L_2	X_M	,, ,, ,, ,,
3	L_2	X_Y	,, ,, ,, ,,
4	L_1	X_Y	,, ,, ,, ,,
5	L_1	X_M	Right end towards left end of L_6.
6	L_2	X_M	,, ,, ,, ,,
7	L_2	X_Y	,, ,, ,, ,,
8	L_1	X_Y	,, ,, ,, ,,

Table II shows the procedure during the first week, which was typical of the other seven weeks. Three observers took part in the optical observations, one of whom made half the total number of determinations, while the other two each made one-quarter of the total number. On Friday of this particular week, étalon L_2 was substituted for L_1, slight adjustments of parallelism and length were applied to étalons L_5 and L_6, and the silica gel in the air-conditioning system was dehydrated for four hours at about 300° C. On Saturday morning the apparatus was closed after all adjustments had been carefully checked, and the toluene-mercury regulators in the room and the apparatus enclosure were readjusted to suit the external temperature conditions. The étalons and air reservoirs were also washed out three times with conditioned air and were then filled in readiness for the observation in air on the following Monday morning. In addition, the total volume of approximately 12 litres of air enclosed in the étalons, reservoirs, and connecting pipe-lines was normally renewed three more times per week, so that weekly dehydration of the silica gel was sufficient to maintain the material at its maximum efficiency.

TABLE II.—Sequence of Observations and Procedure during the First Week.

Arrangement.—Basic Measurement in L_1; End-gauge X_M.

Day.	Morning.			Afternoon.		
	Number of determination.	Condition.	Observer.	Number of determination.	Condition.	Observer.
Monday	I	Air	H. B.	II	Vacuum	R. F. Z.
Tuesday	III	,,	J. E. S.	IV	,,	H. B.
Wednesday . . .	V	,,	H. B.	VI	,,	J. E. S.
Thursday . . .	VII	,,	R. F. Z.	VIII	,,	H. B.
Friday	Apparatus re-arranged.					
Saturday	Apparatus closed.					
Sunday						

It has already been mentioned that the Osram lamp was substituted for the usual Michelson lamp as a source of the standard red radiation of cadmium. A description of the new lamp and of the investigation which led to this change has been published elsewhere (*loc. cit.*). It was shown that the Osram lamp, when excited by a current of 1 ampère, was entirely suitable as an alternative source of the red radiation. In the present work it was found advantageous to increase the exciting current to 1·2 ampère, thereby leading to a considerable increase in the intensity of the red line, without introducing the self-reversal effect which occurs when the current is increased to 2 ampères.

In Table III are given the values of the wave-lengths of the five radiations, both in standard air and in vacuum, which were temporarily accepted for use in the determination of orders of interference by the method of coincidences of excess fractions. The vacuum values were calculated, from those given in air, by use of Pérard's data for the refractive index and dispersion of standard air (*loc. cit.*). The table also includes the numbers of the Wratten filters used to isolate the required radiations.

Table III.—Accepted Wave-lengths in Air and in Vacuum.

Radiation.	Wratten filter No.	λ in Air (1×10^{-10} M).	λ in Vacuum (1×10^{-10} M).
Cadmium red	26	6438·4696*	6440·2493
Cadmium green	55	5085·8212†	5087·2387
Krypton yellow	22	5870·9154‡	5872·5427
Krypton green	77	5570·2892‡	5571·8360
Mercury green	77	5460·7430‡	5462·2605

* Internationally accepted value of Benoît, Fabry, and Perot.

† Sears and Barrell, 'Proc. Roy. Soc.,' A, vol. 139, p. 214 (1933).

‡ Pérard, 'Rev. d'Optique,' vol. 7, p. 1 (1928).

The green line of krypton was chiefly employed, in association with the red line of cadmium, as an auxiliary radiation for the determination of the orders of interference corresponding to the étalons L_1 and L_2. On certain occasions the green line of cadmium was used instead of the krypton line, but the visibility of the interference fringes in the former radiation was markedly inferior to that of the krypton line.

For the determination of the orders of interference corresponding to the difference in length between L_6 and X_M, the green lines of cadmium and mercury were used as auxiliaries. As the mercury green line was unsuitable, owing to its complexity, for the purpose of applying the method of coincidences to the measurement of the much larger difference between L_6 and X_Y, the green line of krypton was used instead for this part of the work. In addition, the yellow line of krypton was used in certain of the preliminary determinations of orders of interference.

Temperatures were measured by means of three platinum thermometers known as Θ, T_1, and T_2. A full description of the thermometers and their calibrations appeared in Appendix II of the previous paper. The thermometer Θ is wound on the longest étalon L_6 and indicated the temperature of this étalon and the internally supported X-gauge. Θ is divided into two halves, called Θ_E and Θ_W, in such a manner that any temperature gradient along the gauge could be observed. The thermometer T_1 was suspended very near to the étalon in which the basic measurement of length was made, while T_2 was suspended near the air reservoirs.

Θ had not been recalibrated since March, 1930, but T_1 and T_2 have been recalibrated frequently since then. Accordingly, before commencing the determinations of X_M and X_Y a comparison was made between Θ, T_1, and T_2, under appropriate conditions in the apparatus enclosure, which showed that no change greater than 0·001° C. had occurred in the calibration of Θ.

It was intended to carry out the determinations of X_M and X_Y as near as possible at 20° C., and for this reason the work was arranged to take place in the spring. Experience had shown that this was the most suitable period of the year for maintaining the apparatus enclosure at 20° C. Owing, however, to the unduly warm weather during the months of April, May, and June in the year 1933 the average temperature of the enclosure had to be raised to about 21° C., the actual values ranging from 20·2° C. at the beginning of the period to 22·1° C. at the end. Reference to the official figures obtained at Kew shows that the average external temperature for these three months was about 2·6° F., or nearly 1·5° C., in excess of normal, which clearly accounted for the high value of the mean temperature at which the enclosure had to be maintained during this period.

The pressure of the residual air in the étalon system during the vacuum determinations was observed by means of a discharge tube pressure indicator (see fig. 2), which was calibrated by means of a McLeod gauge. The discharge was excited by rectified current from a transformer at a potential of about 1000 volts and the width of the dark space gave an indication of the pressure of the residual air. The vacuum pump, which was of the rotary mechanical type enclosed in oil, was capable of reducing the pressure in the étalons to about 0·02 mm. in 20 minutes, and of maintaining this reduced pressure during the period occupied by a set of optical observations in vacuum.

(*c*) *Comparison of the X-gauges with the Composite Gauges.*—The second stage of the procedure for comparing a wave-length of light with a fundamental unit of length is concerned with the determination of the difference between the lengths of an X-gauge and a composite gauge, in terms of the standard wave-length. Representing the metre and yard composite gauges by the symbols M′ and Y′ respectively, then the second stage gives the values of $(M' - X_M)$ and $(Y' - X_Y)$.

For present purposes it is sufficient to recall that M′ and Y′ each consist of an auxiliary end-bar, half an inch shorter than the fundamental standard, which in this stage of the work is associated with one or other of two parallel-faced blocks, each half an inch in thickness, wrung to one of its ends. For ease in identification the two parallel faces of one block are marked 1 and 2, and the similar faces on the other are marked 3 and 4. Since either face of each block may be wrung to either end of the auxiliary bar, there are eight different arrangements of the auxiliary bar and the two blocks. Each arrangement of the composite gauge was compared with the corresponding X-gauge and the mean difference was calculated.

During the series of eight comparisons the gauges were interchanged on their supports in a symmetrical manner, and fig. 1 shows the four dispositions of X_M and M′ for the comparisons in which the block (1, 2) was used. The letters L and R indicate the left and right ends of the gauges.

The end-gauge comparator was found to be extremely susceptible to vibration, and it was not possible to run the thermostatic and stirring arrangements situated inside the enclosure during these comparisons. Furthermore, all observations on the comparator had to be made at times outside the normal working hours of the Laboratory in order to avoid the vibration caused by running machinery in the surrounding buildings.

The observations on the end-gauge comparator were carried out by the three observers already mentioned. Two observations by different observers were made with each arrangement of the gauges to be compared, and in order to preserve the same relation between the three observers as in the determinations of X_M and X_Y, one observer made comparisons for all arrangements of the gauges, while the other two made alternate comparisons.

(*d*) *Comparisons of the Composite Gauges with the Fundamental Standards.*—The final stage of the determination of a fundamental unit of length in terms of a standard wave-length consists of line-standard comparisons, the object of which is to observe the relation between the composite gauges and the fundamental units of length. A complete description of these comparisons is contained in Appendix III. The Prototype Metre and the Imperial Standard Yard are subsequently represented by the symbols M and Y respectively.

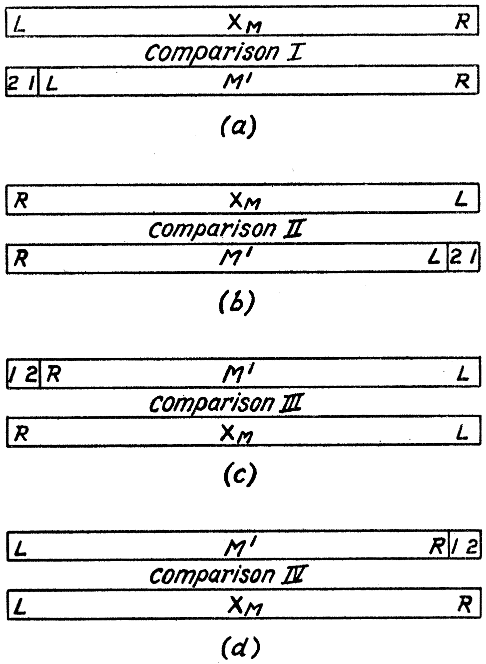

FIG. 1.—Disposition of X_M and M′ with Block (1, 2).

3. *Metre Determinations.*

(*a*) *Results of the Determinations of X_M in Air and in Vacuum.*—The results of the determinations of X_M, in air and in vacuum, in terms of the wave-length, λ_R, of the red radiation of cadmium are given in Tables IV and V respectively. As the air and vacuum determinations were generally made alternately the identification numbers of the former are odd and of the latter even.

In Table IV the temperatures in the column headed T_1 were the readings of the thermometer T_1 and represent the temperatures of the basic étalons. The temperatures in the column headed Θ in both Tables IV and V were the mean readings of the two halves Θ_E and Θ_W, of the thermometer Θ wound on the longest étalon L_6 and represent the temperatures of this étalon and X_M. Since the readings of Θ_E and Θ_W in no case differed by more than 0·003° C. the values of Θ are expressed where necessary to the nearest 0·0005° C. by means of the suffix 5. All temperatures are given in terms of the International Scale.

Pressures in Table IV are expressed in terms of millimetres of mercury at 0° C. under standard gravity, $g = 980{\cdot}665$ cm. per sec. per sec., and are considered to be accurate to within ± 0·02 mm.

Table IV.—Results of the Determinations of X_M in Air.

Observation No.	Observer.	Basic étalon.	T_1 (° C.).	Θ (° C.).	Pressure (mm.).	Observed value of X_M (λ_R).	Correction to mean conditions (λ_R).	Corrected value of X_M (λ_R).
I	H. B.	L_1	20·178	20·177$_5$	760·37	1,553,216·239	+12·397	1,553,228·636
III	J. E. S.	L_1	20·181	20·180$_5$	760·29	1,553,216·047	+12·395	1,553,228·442
V	H. B.	L_1	20·170	20·174$_5$	759·96	1,553,216·064	+12·664	1,553,228·728
VII	R. F. Z.	L_1	20·146	20·155	759·98	1,553,215·636	+12·942	1,553,228·578
IX	J. E. S.	L_2	20·976	20·976	759·80	1,553,227·973	+ 0·554	1,553,228·527
XI	H. B.	L_2	20·988	20·992	759·81	1,553,228·357	+ 0·297	1,553,228·654
XIII	R. F. Z.	L_2	21·106	21·113$_5$	759·23	1,553,229·901	— 1·238	1,553,228·663
XV	H. B.	L_2	21·110	21·113$_5$	760·22	1,553,230·487	— 1·780	1,553,228·707
XVII	R. F. Z.	L_1	21·385	21·394	759·97	1,553,234·622	— 5·926	1,553,228·696
XIX	H. B.	L_1	21·391	21·394	759·98	1,553,234·509	— 5·923	1,553,228·586
XXI	J. E. S.	L_1	21·393	21·402$_5$	759·87	1,553,234·458	— 6·000	1,553,228·458
XXIII	H. B.	L_1	21·406	21·416$_5$	760·01	1,553,234·912	— 6·294	1,553,228·618
XXV	H. B.	L_2	21·397	21·414	759·96	1,553,234·745	— 6·236	1,553,228·509
XXVII	R. F. Z.	L_2	21·411	21·422	760·10	1,553,235·112	— 6·428	1,553,228·684
XXIX	H. B.	L_2	21·424	21·428	760·16	1,553,235·107	— 6·541	1,553,228·566
XXXI	J. E. S.	L_2	21·422	21·428	760·22	1,553,235·154	— 6·578	1,553,228·576
Mean . .			21·005	21·011$_5$	760·00	1,553,228·708		1,553,228·602

Table V.—Results of the Determinations of X_M in Vacuum.

Observation No.	Observer.	Basic étalon.	Θ (° C.).	Observed value of X_M (λ_R).	Correction to mean temperature (λ_R).	Corrected value of X_M (λ_R).
II	R. F. Z.	L_1	20·188$_5$	1,552,795·006	+13·907	1,552,808·913
IV	H. B.	L_1	20·202$_5$	1,552,795·125	+13·674	1,552,808·799
VI	J. E. S.	L_1	20·180	1,552,794·785	+14·049	1,552,808·834
VIII	H. B.	L_1	20·165	1,552,794·564	+14·299	1,552,808·863
X	H. B.	L_2	20·977	1,552,808·139	+ 0·759	1,552,808·898
XII	J. E. S.	L_2	21·114$_5$	1,552,810·449	— 1·535	1,552,808·914
XIV	H. B.	L_2	21·106$_5$	1,552,810·484	— 1·402	1,552,809·082
XVI	R. F. Z.	L_2	21·105	1,552,810·270	— 1·377	1,552,808·893
XVIII	H. B.	L_1	21·400	1,552,815·131	— 6·299	1,552,808·832
XX	R. F. Z.	L_1	21·402	1,552,815·376	— 6·332	1,552,809·044
XXII	J. E. S.	L_1	21·411$_5$	1,552,815·545	— 6·491	1,552,809·054
XXIV	H. B.	L_1	21·415	1,552,815·505	— 6·550	1,552,808·955
XXVI	J. E. S.	L_2	21·413$_5$	1,552,815·444	— 6·524	1,552,808·920
XXVIII	H. B.	L_2	21·427	1,552,815·690	— 6·750	1,552,808·940
XXX	R. F. Z.	L_2	21·423$_5$	1,552,815·620	— 6·692	1,552,808·928
XXXII	H. B.	L_2	21·425	1,552,815·524	— 6·717	1,552,808·807
Mean . .			21·022$_5$			1,552,808·917

The results given in both tables are divided horizontally into four groups, each group comprising the four determinations obtained under one condition of adjustment and arrangement of the apparatus.

(*b*) *Determination of the Fixed Corrections to* X_M.—Before outlining the process for reducing the individual values of X_M to consistent conditions, it is necessary to refer to the fixed corrections to be applied to X_M. A full description of the corrections has already been given in the previous paper (*loc. cit.*, p. 121). The difference between the mean length of the four optical channels of L_6 and its axial length was determined by experiments with the X-gauge removed from the étalon. The measurement was repeated four times and between each measurement the quartz plates were removed from the étalon and then wrung into position again. For each of the four wringings measurements were made by each of the three observers, first with the étalon containing air and then with it evacuated. The results are given in Table VI, which shows the observed values of the correction, in terms of λ_R, to be applied to the observed lengths of the X-gauges to compensate for the difference between the axial length and the mean length of the four channels of L_6, and also the adopted mean values of this correction for determinations both in air and in vacuum.

TABLE VI.—Values of the Correction due to Curvature of the Quartz Plates on L_6.

Observation.	Observer.	Values of correction to X-gauge in terms of λ_R.			
		Air.		Vacuum.	
			(Means.)		(Means.)
1 . . .	J. E. S.	+ 0·018	+ 0·023	− 0·019	− 0·013
	H. B.	+ 0·022		− 0·010	
	R. F. Z.	+ 0·029		− 0·010	
2 . . .	J. E. S.	+ 0·028	+ 0·029	− 0·011	− 0·006
	H. B.	+ 0·026		− 0·004	
	R. F. Z.	+ 0·034		− 0·002	
3 . . .	J. E. S.	+ 0·050	+ 0·047	+ 0·008	+ 0·008
	H. B.	+ 0·047		+ 0·011	
	R. F. Z.	+ 0·044		+ 0·006	
4 . . .	J. E. S.	+ 0·031	+ 0·028	− 0·002	− 0·003
	H. B.	+ 0·026		− 0·006	
	R. F. Z.	+ 0·026		− 0·001	
Mean . .			+ 0·032		− 0·003

It will be noted that the values of the corrections were in good agreement for three of the four conditions of measurement, but in the other condition the corrections were unduly positive. This was possibly due to some slight distortion of the quartz plates during the wringing process. However, the mean values are in good agreement with the values of $+ 0{\cdot}04\ \lambda_R$ and $- 0{\cdot}00_2\ \lambda_R$ determined at the time when the provisional measurements of the metre were made and recorded in the previous paper (*loc. cit.*, p. 123).

The correction due to the effect of the phase change occurring in reflections at a lapped and polished steel surface, which causes the optical length of the X-gauge to be apparently shorter than the mechanical or practical length, has been obtained by a separate investigation on similarly lapped and polished steel surfaces, a description of which has been published elsewhere.* The value there obtained for the difference between the mechanical and optical lengths of an end-gauge, when expressed in terms of the wave-length λ_R, was $0{\cdot}280\ \lambda_R$. This correction is positive when applied to determinations of the optical length in order to derive the mechanical length.

Another correction, due to a similar effect of phase change at the semi-silvered quartz surfaces on L_6, was required to compensate for the somewhat different manner in which these surfaces were used in the determinations of L_6 and $(L_6 - X)$. The correction was determined by the method already described in the previous paper (*loc. cit.*, p. 122), measurements being made by the three observers. Table VII shows the values of the " reflection " correction due to each semi-silvered quartz plate, and the mean value of the total correction which was applied to all measurements of the X-gauges.

TABLE VII.—Values of the " Reflection " Correction in terms of λ_R.

Observer.	Left plate of L_6.	Right plate of L_6.	Total correction.
J. E. S.	+ 0·008	+ 0·013	+ 0·021
,, ...	+ 0·001	+ 0·013	+ 0·014
H. B.	− 0·002	+ 0·002	0·000
R. F. Z. . . .	+ 0·001	+ 0·006	+ 0·007
,, . . .	+ 0·009	+ 0·008	+ 0·017
Mean	$+ 0{\cdot}003_4$	$+ 0{\cdot}008_4$	+ 0·012

Compared with the value of $+ 0{\cdot}044\ \lambda_R$ previously determined for this correction (previous paper, *loc. cit.*, p. 123), the new value was appreciably lower, and this was probably due to the use of denser silver films on the quartz plates for the present work.

Two other fixed corrections were applied to the values of the X-gauges determined in vacuum. The first of these compensated for the elastic expansion of the gauge in vacuum owing to the removal of the atmospheric pressure, and was calculated from measurements of the elastic constants of specimens of the steel bars from which the

---

* ROLT and BARRELL, 'Proc. Roy. Soc.,' A, vol. 122, p. 122 (1929).

gauges were made. These measurements were made in the Engineering Department of the Laboratory, and the mean values of the elastic constants were :—

Young's modulus . $30 \cdot 55 \times 10^6$ lb./sq. in. $\pm\, 0 \cdot 19 \times 10^6$ lb./sq. in.
Poisson's ratio . $0 \cdot 272 \pm 0 \cdot 003$.

Using these values of the constants, the calculated contraction in the length of an X-gauge when taken from vacuum to one atmosphere amounted to 0·000,000,219 of its length. It was estimated that the error associated with this value of the corrective term, due to the variations of the elastic constants quoted above, was ± 0·000,000,004 of the length concerned. The other correction was due to the fact that the vacuum determinations were made at an average residual air pressure of 0·020 mm. Since the residual pressure was the same for all vacuum determinations to within ± 0·003 mm. a flat correction was calculated and applied to the mean values of the X-gauges. The calculated value of the refractive index of air at a pressure of 0·020 mm. is 1·000,000,007, so that the correction amounted to — 7 parts in 10^9 of the lengths of the X-gauges.

The values of the fixed corrections for determinations of X_M in air were therefore :—

(i) Correction to axial length of L_6 . . . . $= +\,0 \cdot 032\ \lambda_R$
(ii) Correction to mechanical length of X_M . . . . $= +\,0 \cdot 280$
(iii) "Reflection" correction . . . . $= +\,0 \cdot 012$
Total correction for air determinations . . . . . $= +\,0 \cdot 324\ \lambda_R$

The values of the fixed corrections for determinations of X_M in vacuum were :—

(i) Correction to axial length of L_6 . . . . $= -\,0 \cdot 003\ \lambda_R$
(ii) Correction to mechanical length of X_M . . . . $= +\,0 \cdot 280$
(iii) "Reflection" correction . . . . $= +\,0 \cdot 012$
(iv) Correction to 1 atmosphere ($-\,0 \cdot 219\ X_M \times 10^{-6}$) . . . $= -\,0 \cdot 340$
(v) Correction for residual pressure ($-\,0 \cdot 007\ X_M \times 10^{-6}$) . $= -\,0 \cdot 011$
Total correction for vacuum determinations . . . $= -\,0 \cdot 062\ \lambda_R$

(*c*) *Reduction of Values of* X_M *to the Mean Conditions of Observation.*—The reduction of the individual values of X_M to values corresponding to the mean conditions of observation was readily performed in the vacuum determinations, for it only required a knowledge of the coefficient of thermal expansion of X_M. The coefficient of X_M was measured in a line-standard comparator for a temperature range extending from 0° C. to 30° C. Fine lines were ruled on one of the webs of the gauge and the usual microscope comparisons were made against a line-standard of known coefficient in the bath of the comparator. The expansion formula for the bar X_M was found to be :—

$$L_t = L_0\,[1 + (10 \cdot 494t + 0 \cdot 00595t^2)\,10^{-6}]$$

and the coefficient of expansion at a temperature t° C. was given by the expression :—

$$(10 \cdot 494 + 0 \cdot 00595 \times 2t)\,10^{-6}.$$

The reduction of the determinations in air further involved a knowledge of the refractive index of air and of its variation with temperature and pressure. Additional complications arose from the fact that the temperatures indicated by T_1 and Θ were generally different, though only to the extent of $0 \cdot 006_5$° C. on the average of the 16 determinations.

The method adopted for the reduction of the determinations in air cannot be properly described without some reference to the experimental procedure. Each value of X_M was obtained by deriving the difference between the observed value of L_6 and the observed value of $(L_6 - X_M)$. The value of L_6 was determined by optical multiplication from one of the basic étalons, L_1 or L_2, and was expressed, therefore, in terms of waves in air under the conditions of temperature and pressure existing in the basic étalon, whereas the value of $(L_6 - X_M)$ was determined by direct measurement in terms of waves in air under the conditions of temperature and pressure existing in L_6. It was assumed that the pressure of the air enclosed in the étalon system was uniform.

It will be seen from Table IV that the mean temperature of the basic étalons was 21·005° C., the mean temperature of L_6 and X_M was $21 \cdot 011_5$° C., where both mean temperatures have been evaluated to the nearest 0·0005° C., and the mean pressure was 760·00 mm. Since the fundamental measurement of length was made in terms of waves in air enclosed within the basic étalons, it was decided to adopt the temperature of 21·005° C. as the basis for purposes of reduction.

In a particular determination of X_M, let N_a be the number of waves, of wave-length λ_a, contained in L_6, where λ_a is the wave-length in air at a temperature of T_1° C. and a pressure of h mm. Also let n'_a be the number of waves, of wave-length λ'_a, contained in $(L_6 - X_M)$, where λ'_a is the wave-length in air at a temperature of Θ° C. and a pressure of h mm. Similarly, let N_m and n_m be the numbers of waves, of length λ_m, contained in L_6 and in $(L_6 - X_M)$ respectively, where λ_m is the wave-length in air under the mean conditions of 21·005° C. and 760·00 mm. If μ_a is the refractive index of air at T_1° C. and h mm., μ'_a is the refractive index of air at Θ° C. and h mm., and μ_m is the refractive index of air at 21·005° C. and 760·00 mm., then, since $N_a\lambda_a = N_m\lambda_m$ and $\lambda_a\mu_a = \lambda_m\mu_m$,

$$N_a = N_m \frac{\mu_a}{\mu_m}, \quad \ldots \ldots \ldots \ldots \quad (1)$$

and similarly

$$n'_a = n_m \frac{\mu'_a}{\mu_m}. \quad \ldots \ldots \ldots \ldots \quad (2)$$

But

$$(\mu_a - 1) = (\mu_0 - 1) \cdot \frac{h}{760} \cdot \frac{1}{1 + \alpha T_1},$$

$$(\mu'_a - 1) = (\mu_0 - 1) \cdot \frac{h}{760} \cdot \frac{1}{1 + \alpha\Theta},$$

and

$$(\mu_m - 1) = (\mu_0 - 1) \cdot \frac{1}{1 + 21 \cdot 005\alpha},$$

where μ_0 is the refractive index of air at 0° C. and 760 mm. The value of α was assumed to be 0·003716, as determined by PÉRARD (*loc. cit.*, p. 77). Substituting these expressions for the refractive index in equations (1) and (2), then

$$N_a = N_m\left(1 - A + B \cdot \frac{h}{1+\alpha T_1}\right), \quad \ldots \ldots \ldots \quad (3)$$

$$n'_a = n_m\left(1 - A + B \cdot \frac{h}{1+\alpha\Theta}\right), \quad \ldots \ldots \ldots \quad (4)$$

where

$$A = \frac{\mu_0 - 1}{\mu_0 + 21\cdot005\alpha} \quad \text{and} \quad B = \frac{(\mu_0 - 1)(1 + 21\cdot005\alpha)}{760\,(\mu_0 + 21\cdot005\alpha)}.$$

As the bracketed expressions in equations (3) and (4) only differed from unity by a few parts in a million in the present experiments, these equations may be re-written in the following form :—

$$N_m = N_a\left(1 + A - B \cdot \frac{h}{1+\alpha T_1}\right), \quad \ldots \ldots \ldots \quad (5)$$

$$n_m = n'_a\left(1 + A - B \cdot \frac{h}{1+\alpha\Theta}\right) \quad \ldots \ldots \ldots \quad (6)$$

The corrected value of X_M was then given by $(N_m - n_m)$.

The values of A and B in equations (5) and (6) can be determined if the refractive index of air is known. Since the determinations of X_M were made both in air and in vacuum, it was possible to derive the refractive index from the observations themselves in the manner now to be described. Referring to Table IV, the arithmetical mean of the observed values of X_M in air was 1,553,228·708 λ_R, corresponding to a mean temperature by T_1 of 21·005° C., a mean temperature by Θ of 21·011$_5$° C., and a mean pressure of 760·00 mm. The correction to X_M due to the difference of refractive index of air at temperatures of 21·005° C. and 21·011$_5$° C. respectively was calculated by use of equation (6), where $(\mu_0 - 1)$ was assumed to have the value of $291\cdot77 \times 10^{-6}$, derived from PÉRARD's data (*loc. cit.*, p. 78), and n'_a was approximately 10,000 λ_R. It was found that $(n_m - n'_a) = +\,10{,}000 \times 0\cdot01 \times 10^{-6} = +\,0\cdot0001\ \lambda_R$, so that this correction was entirely negligible. The correction necessary to reduce the value of X_M, determined at a mean temperature of 21·011$_5$° C., to a value corresponding to the temperature basis of 21·005° C. was derived in the usual manner from the expansion formula already given, and was found to be — 0·108 λ_R for the temperature difference of — 0·006$_5$° C. Therefore the mean optical length of X_M in air, containing no moisture nor carbon dioxide, at 21·005° C. and 760·00 mm. was 1,553,228·600 λ_R, which, when increased by an amount 0·324 λ_R due to the fixed corrections, gave a value of 1,553,228·924 λ_R.

As shown in Table V, the mean corrected value of X_M in vacuum at the mean temperature of 21·022$_5$° C. was 1,552,808·917 λ_R. The reduction to the temperature basis of 21·005° C. introduced a correction of $-0\cdot292\ \lambda_R$, which, combined with the value of $-0\cdot062\ \lambda_R$ due to the fixed corrections, gave a total correction of $-0\cdot354\ \lambda_R$. Therefore the value of X_M at 21·005° C., in terms of λ_R in vacuum, was 1,552,808·563 λ_R.

As μ_m is the refractive index of air, at 21·005° C. and 760·00 mm., containing no moisture nor carbon dioxide, then—

$$\mu_m = \frac{1{,}553{,}228\cdot924}{1{,}552{,}808\cdot563} = 1\cdot000{,}270{,}710.$$

But

$$(\mu_m - 1) = \frac{\mu_0 - 1}{1 + 21\cdot005\alpha},$$

and therefore $(\mu_0 - 1) = 270\cdot710 \times 1\cdot078{,}055 \times 10^{-6} = 291\cdot840 \times 10^{-6}$. This value of $(\mu_0 - 1)$ was used to calculate the constants A and B in equations (5) and (6). Equations (5) and (6) may then be re-written in this manner :—

$$(N_m - N_a) = N_a\left(270\cdot637 - 0\cdot383{,}896\frac{h}{1 + \alpha T_1}\right)10^{-6}, \quad \ldots\ldots \quad (7)$$

$$(n_m - n'_a) = n'_a\left(270\cdot637 - 0\cdot383{,}896\frac{h}{1 + \alpha\Theta}\right)10^{-6}, \quad \ldots\ldots \quad (8)$$

where $\alpha = 0\cdot003716$, and the approximate values of N_a and n'_a to be substituted on the right-hand sides of these equations are $N_a = 1\cdot5633 \times 10^6$ and $n'_a = 0\cdot0100 \times 10^6$.

The corrections to the mean conditions, shown in Table IV, result from the combination of the values of the corrections to L_6 and $(L_6 - X_M)$ calculated from equations (7) and (8), with the values of the corrections due to the coefficient of expansion of X_M. Therefore the corrected values of X_M shown in the last column of Table IV were the values of X_M at 21·005° C. in terms of waves in air, at 21·005° C. and 760·00 mm., containing no moisture nor carbon dioxide. The mean corrected value of X_M shown at the foot of the column is 1,553,228·602 λ_R and differs by only 0·002 λ_R from that calculated from the mean of the observed values. Further application of the method of successive approximation could be made by utilising this new value of X_M to determine an improved value of $(\mu_0 - 1)$, thereby obtaining new values of the corrections to compensate for the variations of refractive index. But it is obvious that such application would not appreciably alter the present mean value of X_M, which was therefore taken as final.

The determinations of X_M may be summarised as follows :—

In Air.—If λ_R be the wave-length of the cadmium red radiation in air, at 21·005° C. and 760·00 mm., containing no moisture nor carbon dioxide :—

X_M at 21·005° C.	= 1,553,228·602 λ_R
Fixed correction for air determinations	= + 0·324
Mechanical length of X_M at 21·005° C.	= 1,553,228·926 λ_R

In Vacuum.—If λ_R be the wave-length of the cadmium red radiation in vacuum :—

X_M at $21\cdot022_5$° C. . . . . $= 1{,}552{,}808\cdot917\ \lambda_R$

Correction to $21\cdot005$° C. . . . . $= \quad -0\cdot292$

Fixed correction for vacuum determinations . . . . $= \quad -0\cdot062$

Mechanical length of X_M at $21\cdot005$° C. . . . . $= 1{,}552{,}808\cdot563\ \lambda_R$

Refractive index of air at $21\cdot005$° C., etc. . . . . $= \dfrac{1{,}553{,}228\cdot926}{1{,}552{,}808\cdot563}$

$= 1\cdot000{,}270{,}711,$

whence

$$(\mu_0 - 1) = 270\cdot711\ (1 + 21\cdot005\ \alpha)\ 10^{-6} = 291\cdot841 \times 10^{-6}.$$

(*d*) *Results of the Determinations of* ($M' - X_M$).—The results of the comparisons of X_M with the composite gauge M' are given in Table VIII. The comparisons identified by the numbers I to IV were those in which block (1, 2) entered, and the particular dispositions of the two gauges corresponding to the four comparisons are shown in the

TABLE VIII.—Results of the determinations of ($M' - X_M$).

No. of Com-parison.	Observer.	Temperature of comparison. T_1 (° C.).	T_2 (° C.).	Mean (° C.).	Observed value of ($M' - X_M$) (λ_R).	Correction to 21·005° C. (λ_R).	Corrected value of ($M' - X_M$) (λ_R).
I	(*a*) H. B. . .	22·307	22·316	22·312	9·515	− 0·164	9·351
	(*b*) R. F. Z. .	22·295	22·304	22·300	9·469	− 0·163	9·306
II	(*a*) J. E. S. .	21·860	21·866	21·863	9·394	− 0·108	9·286
	(*b*) H. B. . .	21·867	21·877	21·872	9·376	− 0·109	9·267
III	(*a*) R. F. Z. .	22·212	22·221	22·216	9·354	− 0·154	9·200
	(*b*) H. B. . .	22·222	22·232	22·227	9·433	− 0·156	9·277
IV	(*a*) H. B. . .	22·202	22·212	22·207	9·318	− 0·153	9·165
	(*b*) J. E. S. .	22·221	22·234	22·228	9·422	− 0·156	9·266
V	(*a*) J. E. S. .	22·416	22·394	22·405	9·431	− 0·178	9·253
	(*b*) H. B. . .	22·423	22·396	22·410	9·499	− 0·179	9·320
VI	(*a*) H. B. . .	22·140	22·144	22·142	9·411	− 0·143	9·268
	(*b*) R. F. Z. .	22·144	22·148	22·146	9·444	− 0·144	9·300
VII	(*a*) H. B. . .	21·762	21·774	21·768	9·372	− 0·097	9·275
	(*b*) J. E. S. .	21·769	21·779	21·774	9·363	− 0·098	9·265
VIII	(*a*) R. F. Z. .	22·396	22·403	22·400	9·416	− 0·178	9·238
	(*b*) H. B. . .	22·406	22·410	22·408	9·405	− 0·179	9·226
						Mean . .	9·266

diagram, fig. 1. With block (3, 4) substituted for block (1, 2), the same sequence of arrangements of the gauges was followed and the comparisons identified by the numbers V to VIII. It has already been mentioned that one observer took part in every comparison, the other two taking part in alternate comparisons, so that for each arrangement the value of $(M' - X_M)$ was obtained by two different observers, with no disturbance of the apparatus between the two determinations. The work was arranged so that the observations of any one person would give a definitive value of $(M' - X_M)$.

As no stirring of the air was allowable either before or during these observations, it was possible that horizontal gradients of temperature, of a variable nature, might be established in a direction at right angles to the axes of the gauges. Therefore, in order to compensate in some measure for this possible effect, the first observer commenced his observations on the ends of X_M, while the second commenced his on the ends of M'. It is believed the larger discrepancies between the values of $(M' - X_M)$ obtained by the (*a*) and (*b*) observers were chiefly due to this cross-gradient of temperature and not to errors of observation. It has been decided to undertake certain alterations to the comparator which it is hoped may overcome the difficulties due to vibration, and so enable the thermostatic control to be used in future comparisons of this kind. It is considered, however, that by the arrangement of observations adopted in the present comparisons the mean values have been satisfactorily established, though the work was not carried out under the most advantageous conditions.

The temperatures given in columns headed T_1 and T_2 in Table VIII were the readings of the two platinum resistance thermometers T_1 and T_2 respectively, which were placed with their resistance " bulbs " in close proximity with the gauges and near the positions of the Airy bands of the latter. The mean values of temperature shown in Table VIII are the means of the readings of T_1 and T_2.

The results of the comparisons are expressed in terms of the difference $(M' - X_M)$, which is itself given in terms of the wave-length λ_R of the red radiation of cadmium in air. All observed values of $(M' - X_M)$ were reduced to the temperature basis of 21·005° C. by means of the corrections, given in the penultimate column, which were calculated by use of the relative coefficient of thermal expansion of the gauges derived from the expansion formulæ of the gauges. The relevant data for M' is given in Appendix III. The mean value of $(M' - X_M)$ at 21·005° C. was $9{\cdot}266_4\ \lambda_R$. If the value of the refractive index of air is assumed to be 1·00027 then the calculated value of $(M' - X_M)$ at 21·005° C. in terms of λ_R in vacuum is $9{\cdot}263_9\ \lambda_R$. The values finally adopted as the result of the comparisons of X_M with M' were :—

$$(M' - X_M) \text{ at } 21{\cdot}005^\circ \text{ C.} = 9{\cdot}266\ \lambda_R \text{ in air.}$$
$$= 9{\cdot}264\ \lambda_R \text{ in vacuum.}$$

(*e*) *Result of the Comparison of* M' *and* M.—The comparison of the composite gauge M' with the Prototype Metre was carried out by the usual metrological procedure, and an account of the work is given in Appendix III. Such comparisons as were made at the

National Physical Laboratory were done by three other observers more accustomed to this class of work. The result of the comparison was :—

$$M' \text{ at } 20^\circ \text{ C.} = 1{\cdot}000{,}042{,}78 \text{ M.}$$

Using the value of the coefficient of thermal expansion of M′ derived from the data given in Appendix III, the value of the length of M′ at the temperature basis of 21·005° C. was calculated. Thus, for a temperature change of + 1·005° C., the correction was $+ 10{\cdot}87 \text{ M} \times 10^{-6}$, so that :—

$$M' \text{ at } 21{\cdot}005^\circ \text{ C.} = 1{\cdot}000{,}053{,}65 \text{ M.}$$

The final calculation of the value of the Prototype Metre in terms of the wave-length of the red radiation of cadmium is deferred to section 6 of this paper.

4. *Yard Determinations.*

(*a*) *Results of the Determinations of* X_Y *in Air and in Vacuum.*—The results of the determinations of X_Y in terms of the wave-length, λ_R, of the red radiation of cadmium are given in Tables IX and X, where the information is presented in exactly the same manner as for the determinations of X_M in Tables IV and V.

TABLE IX.—Results of the Determinations of X_Y in Air.

Observation No.	Observer.	Basic étalon.	T_1 (° C.).	Θ (° C.).	Pressure (mm.).	Observed value of X_Y (λ_R).	Correction to mean conditions (λ_R).	Corrected value of X_Y (λ_R).
I	J. E. S.	L_2	20·689	20·692	760·09	1,420,218·633	+ 9·127	1,420,227·760
III	H. B.	L_2	20·770	20·762	759·74	1,420,219·505	+ 8·346	1,420,227·851
V	R. F. Z.	L_2	20·771	20·768	759·94	1,420,219·663	+ 8·155	1,420,227·818
VII	H. B.	L_2	20·778	$20{\cdot}779_5$	760·02	1,420,219·985	+ 7·947	1,420,227·932
IX	H. B.	L_1	20·380	20·389	760·18	1,420,214·667	+13·284	1,420,227·951
XI	R. F. Z.	L_1	20·374	20·377	759·96	1,420,214·414	+13·572	1,420,227·986
XIII	H. B.	L_1	20·372	$20{\cdot}375_5$	759·99	1,420,214·278	+13·578	1,420,227·856
XV	J. E. S.	L_1	20·385	20·379	760·05	1,420,214·419	+13·510	1,420,227·929
XVII	H. B.	L_2	22·155	$22{\cdot}137_5$	759·97	1,420,238·775	—10·909	1,420,227·866
XIX	J. E. S.	L_2	22·165	$22{\cdot}147_5$	759·94	1,420,238·849	—11·034	1,420,227·815
XXI	R. F. Z.	L_2	22·144	22·141	760·10	1,420,238·909	—11·045	1,420,227·864
XXIII	H. B.	L_2	22·134	22·133	759·97	1,420,238·798	—10·870	1,420,227·928
XXV	R. F. Z.	L_1	22·130	$22{\cdot}126_5$	760·11	1,420,238·890	—10·847	1,420,228·043
XXVII	H. B.	L_1	22·125	$22{\cdot}121_5$	760·03	1,420,238·689	—10·738	1,420,227·951
XXIX	J. E. S.	L_1	22·123	22·120	759·85	1,420,238·467	—10·627	1,420,227·840
XXXI	H. B.	L_1	22·128	$22{\cdot}123_5$	760·08	1,420,238·620	—10·788	1,420,227·832
Mean . .			$21{\cdot}351_5$	$21{\cdot}348_5$	760·00	1,420,227·848		1,420,227·889

TABLE X.—Results of the Determinations of X_Y in Vacuum.

Observation No.	Observer.	Basic étalon.	Θ (° C.).	Observed value of X_Y (λ_R).	Correction to mean temperature (λ_R).	Corrected value of X_Y (λ_R).
II	H. B.	L_2	$20\cdot689_5$	1,419,834·252	+ 9·987	1,419,844·239
IV	R. F. Z.	L_2	$20\cdot765$	1,419,835·289	+ 8·837	1,419,844·126
VI	H. B.	L_2	$20\cdot768_5$	1,419,835·385	+ 8·784	1,419,844·169
VIII	J. E. S.	L_2	$20\cdot781_5$	1,419,835·584	+ 8·586	1,419,844·170
X	J. E. S.	L_1	$20\cdot388$	1,419,829·550	+14·579	1,419,844·129
XII	H. B.	L_1	$20\cdot372$	1,419,829·418	+14·822	1,419,844·240
XIV	R. F. Z.	L_1	$20\cdot374_5$	1,419,829·465	+14·784	1,419,844·249
XVI	H. B.	L_1	$20\cdot386_5$	1,419,829·598	+14·601	1,419,844·199
XVIII	R. F. Z.	L_2	$22\cdot103_5$	1,419,855·746	—11·563	1,419,844·183
XX	H. B.	L_2	$22\cdot114_5$	1,419,856·019	—11·731	1,419,844·288
XXII	H. B.	L_2	$22\cdot143$	1,419,856·432	—12·165	1,419,844·267
XXIV	J. E. S.	L_2	$22\cdot130$	1,419,856·185	—11·967	1,419,844·218
XXVI	H. B.	L_1	$22\cdot128$	1,419,856·276	—11·937	1,419,844·339
XXVIII	J. E. S.	L_1	$22\cdot126$	1,419,856·007	—11·906	1,419,844·101
XXX	H. B.	L_1	$22\cdot121_5$	1,419,856·030	—11·838	1,419,844·192
XXXII	R. F. Z.	L_1	$22\cdot124_5$	1,419,856·160	—11·883	1,419,844·277
Mean . .			$21\cdot345$			1,419,844·212

(*b*) *Fixed Corrections to* X_Y.—For the determinations of X_Y in air, the values of the fixed corrections remained the same as those already quoted for the determinations of X_M in air, and the total correction was, therefore, $+ 0\cdot324\ \lambda_R$. For the determinations in vacuum, two items in the list quoted for X_M were slightly changed, viz., the correction to the condition of immersion in one atmosphere was reduced from $- 0\cdot340\ \lambda_R$ to $- 0\cdot311\ \lambda_R$, and the correction for the residual air pressure of 0·020 mm. was reduced from $- 0\cdot011\ \lambda_R$ to $- 0\cdot010\ \lambda_R$, since both corrections depended upon the length of the X-gauge. Therefore the values of the fixed corrections for determinations of X_Y in vacuum were :—

(i) Correction to axial length of L_6 . . . . $= - 0\cdot003\ \lambda_R$
(ii) Correction to mechanical length of X_Y . . . . $= + 0\cdot280$
(iii) " Reflection " correction . . . . $= + 0\cdot012$
(iv) Correction to 1 atmosphere ($- 0\cdot219\ X_Y \times 10^{-6}$) . . . . $= - 0\cdot311$
(v) Correction for residual pressure ($- 0\cdot007\ X_Y \times 10^{-6}$) . . . $= - 0\cdot010$
Total correction for vacuum determinations . . . . $= - 0\cdot032\ \lambda_R$

(*c*) *Reduction of Values of* X_Y *to the Mean Conditions of Observation.*—The reduction of the individual values of X_Y to the mean conditions of observation was made by the same process as that already described for X_M in section 3 (*c*). From Table IX it will be seen that the mean observed value of X_Y was 1,420,227·848 λ_R, corresponding to a mean temperature by T_1 of $21 \cdot 351_5°$ C., a mean temperature by Θ of $21 \cdot 348_5°$ C. and a mean pressure of 760·00 mm., where the means by T_1 and Θ have been evaluated to the nearest 0·0005° C. The equations for the reduction of the values of L_6 and $(L_6 - X_Y)$ to the mean conditions of observation are similar to equations (5) and (6) of section **3** (*c*), where N_m and N_a still refer to L_6, but n_m and n'_a now refer to $(L_6 - X_Y)$, and the suffix "*m*" refers to air, at $21 \cdot 351_5°$ C. and 760·00 mm., containing no moisture nor carbon dioxide. The values of A and B in equations (5) and (6) under these circumstances are given by the following expressions :—

$$A = \frac{\mu_0 - 1}{\mu_0 + 21 \cdot 351_5 \alpha}, \quad B = \frac{(\mu_0 - 1)\,(1 + 21 \cdot 351_5 \alpha)}{760\,(\mu_0 + 21 \cdot 351_5 \alpha)},$$

where α is assumed to have Pérard's value of 0·003716.

The correction to X_Y due to the difference of refractive index of air at temperatures of $21 \cdot 351_5°$ C. and $21 \cdot 348_5°$ C. respectively was calculated by use of the modified equation (6), in which $(\mu_0 - 1)$ was provisionally assumed to have the value of $291 \cdot 841 \times 10^{-6}$, derived from the determinations of X_M, and n'_a was approximately 143,000 λ_R. It was found that the correction to $(L_6 - X_Y)$, due to refractive index only, was $-$ 0·0004 λ_R, which was just small enough to be neglected in the ensuing calculations.

The correction necessary to increase the value of X_Y, determined at a mean temperature of $21 \cdot 348_5°$ C., to a value corresponding to the temperature basis of $21 \cdot 351_5°$ C. was derived in the usual manner from the expansion formula for X_Y. The expansion formula for X_Y was obtained from measurements in a line-standard comparator over the same temperature range and in the same manner as for X_M, and was :—

$$L_t = L_0\,[1 + (10 \cdot 531 t + 0 \cdot 00474\, t^2)]\, 10^{-6},$$

whence the coefficient of expansion at a temperature $t°$ C. was given by the expression :—

$$(10 \cdot 531 + 0 \cdot 00474 \times 2t)\, 10^{-6}.$$

The correction for a temperature change of + 0·003° C. was therefore + 0·046 λ_R, which, augmented by the fixed correction in air of 0·324 λ_R resulted in a total correction of + 0·370 λ_R. Thus the corrected mean observed value of X_Y at $21 \cdot 351_5°$ C. in terms of waves in air at $21 \cdot 351_5°$ C. and 760·00 mm., containing no moisture nor carbon dioxide, was 1,420,228·218 λ_R.

In Table X the corrections to the mean observed temperature of 21·345° C. were determined by use of the coefficient of thermal expansion of X_Y already quoted, and it will be seen that the mean value of X_Y at 21·345° C. in vacuum was 1,419,844·212 λ_R. The calculated correction to the temperature of $21 \cdot 351_5°$ C. was + 0·099 λ_R, which, combined with the value of $-$ 0·032 λ_R due to the fixed corrections, gave a total correction of + 0·067 λ_R. Therefore the mean value of X_Y at $21 \cdot 351_5°$ C. in terms of λ_R in vacuum was 1,419,844·279 λ_R.

If μ_m be the refractive index of air at $21 \cdot 351_5$° C. and 760·00 mm., containing no moisture nor carbon dioxide, then :—

$$\mu_m = \frac{1{,}420{,}228 \cdot 218}{1{,}419{,}844 \cdot 279} = 1 \cdot 000{,}270{,}409,$$

whence $(\mu_0 - 1) = 270 \cdot 409 \; (1 + 21 \cdot 351_5 \, \alpha) \times 10^{-6} = 291 \cdot 864 \times 10^{-6}$, $A = 270 \cdot 336 \times 10^{-6}$ and $B = 0 \cdot 383{,}928 \times 10^{-6}$.

Since $N_a = 1 \cdot 5633 \times 10^6$ and $n'_a = 0 \cdot 1430 \times 10^6$, the expressions for the corrections $(N_m - N_a)$ and $(n_m - n'_a)$, to be applied to L_6 and $(L_6 - X_Y)$ respectively for the reduction to mean conditions were :—

$$(N_m - N_a) = 1 \cdot 5633 \left(270 \cdot 336 - 0 \cdot 383{,}928 \, \frac{h}{1 + \alpha T_1}\right) \quad . \; . \; . \; . \; . \; . \; (9)$$

$$(n_m - n'_a) = 0 \cdot 1430 \left(270 \cdot 336 - 0 \cdot 383{,}928 \, \frac{h}{1 + \alpha \Theta}\right) \quad . \; . \; . \; . \; . \; (10)$$

The corrections to the mean conditions shown in Table IX result from the combination of corrections, calculated by means of equations (9) and (10), with the corrections due to the coefficient of expansion of X_Y, and therefore the corrected values of X_Y shown in the last column of Table IX are the values of X_Y at $21 \cdot 351_5$° C. in terms of waves in air, at $21 \cdot 351_5$° C. and 760·00 mm., containing no moisture nor carbon dioxide. The mean corrected value of X_Y shown at the foot of the column is 1,420,227·889 λ_R, which differs by 0·005 λ_R from that calculated from the mean of the observed values. As with X_M, further application of the method of successive approximation was unnecessary, and the value of 1,420,227·889 λ_R was accepted as final.

The results of the determinations of X_Y may be summarized as follows :—

In Air.—If λ_R be the wave-length of the cadmium red radiation in air, at $21 \cdot 351_5$° C. and 760·00 mm., containing no moisture nor carbon dioxide :—

X_Y at $21 \cdot 351_5$° C. = 1,420,227·889 λ_R
Fixed correction for air determinations . . . . = + 0·324
Mechanical length of X_Y at $21 \cdot 351_5$° C. . . . . = 1,420,228·213 λ_R

In Vacuum.—If λ_R be the wave-length of the cadmium red radiation in vacuum :—

X_Y at 21·345° C. . . . . = 1,419,844·212 λ_R
Correction to $21 \cdot 351_5$° C. . . . . = + 0·099
Fixed correction for vacuum determinations . . . . . . = − 0·032
Mechanical length of X_Y at $21 \cdot 351_5$° C. . . . . = 1,419,844·279 λ_R
Refractive index of air at $21 \cdot 351_5$° C., etc. . . . . $= \dfrac{1{,}420{,}228 \cdot 213}{1{,}419{,}844 \cdot 279}$
$= 1 \cdot 000{,}270{,}406,$

whence $(\mu_0 - 1) = 270 \cdot 406 \; (1 + 21 \cdot 351_5 \, \alpha) \, 10^{-6} = 291 \cdot 861 \times 10^{-6}$.

(*d*) *Results of the Determinations of* $(Y' - X_Y)$.—The results of the comparisons of X_Y with the composite gauge Y' are given in Table XI, where the information is presented in exactly the same manner as that for the comparison of X_M with M' shown in Table VIII. The two ruled blocks were the same as those used in the metre determinations.

TABLE XI.—Results of the Determinations of $(Y' - X_Y)$.

No. of comparison.	Observer.	Temperature of comparison.			Observed value of $(Y' - X_Y)$. (λ_R).	Correction to $21\cdot351_5$° C. (λ_R).	Corrected value of $(Y' - X_Y)$. (λ_R).
		T_1 (° C.).	T_2 (° C.).	Mean (° C.).			
I	(*a*) H. B. . .	21·744	21·754	21·749	44·085	— 0·004	44·081
	(*b*) R. F. Z. .	21·732	21·743	21·738	43·906	— 0·004	43·902
II	(*a*) J. E. S. .	21·825	21·835	21·830	44·012	— 0·004	44·008
	(*b*) H. B. . .	21·841	21·853	21·847	44·051	— 0·004	44·047
III	(*a*) R. F. Z. .	21·797	21·786	21·792	43·897	— 0·004	43·893
	(*b*) H. B. . .	21·790	21·778	21·784	43·894	— 0·004	43·890
IV	(*a*) H. B. . .	21·831	21·842	21·836	44·054	— 0·004	44·050
	(*b*) J. E. S. .	21·830	21·840	21·835	44·050	— 0·004	44·046
V	(*a*) H. B. . .	22·000	22·010	22·005	44·126	— 0·006	44·120
	(*b*) J. E. S. .	21·985	21·999	21·992	44·100	— 0·005	44·095
VI	(*a*) R. F. Z. .	21·693	21·721	21·707	43·977	— 0·004	43·973
	(*b*) H. B. . .	21·679	21·708	21·694	44·001	— 0·003	43·998
VII	(*a*) J. E. S. .	21·784	21·796	21·790	44·117	— 0·004	44·113
	(*b*) H. B. . .	21·797	21·809	21·803	44·108	— 0·004	44·104
VIII	(*a*) H. B. . .	21·779	21·790	21·784	44·119	— 0·004	44·115
	(*b*) R. F. Z. .	21·813	21·827	21·820	44·098	— 0·004	44·094
						Mean . .	44·033

All the observed values of $(Y' - Y_Y)$ shown in Table XI were reduced to the temperature basis of $21\cdot351_5$° C. by means of the corrections, shown in the penultimate column, which were calculated by using the relative coefficient of thermal expansion derived from the expansion formulæ of Y' and X_Y. The relevant data for Y' is given in Appendix III. The mean value of $(Y' - X_Y)$ at $21\cdot351_5$° C. was $44\cdot033_1$ λ_R, where λ_R is the wave-length in air. If the value of the refractive index of air is assumed to be 1·00027, then the calculated value of $(Y' - X_Y)$ at $21\cdot351_5$° C., in terms of λ_R in vacuum, is $44\cdot021_2$ λ_R. The values finally accepted as the result of the comparisons of X_Y with Y' were :—

$$(Y' - X_Y) \text{ at } 21\cdot351_5\text{° C.} = 44\cdot033\ \lambda_R \text{ in air}$$
$$= 44\cdot021\ \lambda_R \text{ in vacuum.}$$

(*e*) *Result of the Comparison of* Y′ *and* Y.—A description of the comparison of the composite yard gauge Y′ with the Imperial Standard Yard Y is given, together with the result obtained, in Appendix III. The result of the comparison was :—

$$\text{Y}' \text{ at } 62^\circ \text{ F. } (16 \cdot 666_7^\circ \text{ C.}) = 0 \cdot 999{,}999{,}11 \text{ Y.}$$

Using the value of the thermal coefficient of expansion of Y′ derived from the data quoted in Appendix III, the value of the length of Y′ at the temperature basis of $21 \cdot 351_5^\circ$ C. was calculated. Thus, for a temperature change of $+ 4 \cdot 684_8^\circ$ C. the correction was $+ 50 \cdot 19 \text{ Y} \times 10^{-6}$, so that :—

$$\text{Y}' \text{ at } 21 \cdot 351_5^\circ \text{ C.} = 1 \cdot 000{,}049{,}30 \text{ Y.}$$

The final calculation of the value of Y in terms of the wave-length of the red radiation of cadmium is deferred to section 6 of this paper.

5. *Discussion of Errors.*

(*a*) *Determinations of* X_M *and* X_Y.—The analysis of the errors of the complete determinations of the metre and the yard in terms of wave-lengths of light is best performed in three stages, corresponding with the three major operations into which the complete determinations are subdivided. Considering first, then, the determinations of X_M and X_Y, it is of interest to compare the values of the X-gauges obtained from the two basic étalons and by the three observers. This comparison is displayed in Tables XII, XIII, XIV, and XV for the determinations of X_M in air, X_M in vacuum, X_Y in air, and X_Y in vacuum respectively.

The values in the columns headed L_1 and L_2 in the four tables have been taken respectively from the columns of corrected values of the X-gauges given in Tables IV, V, IX, and X. The values at the foot of these two columns in each of the tables represent respectively the mean values of the X-gauges obtained by the three observers from L_1 and L_2. The values in the " Mean " columns are the mean values obtained from both basic étalons by each observer and the values at the foot of these columns are the grand means obtained from both basic étalons by the three observers. It should be noted that, in the calculation of grand means, the mean values obtained by the observer H. B. were assigned twice the weight of the mean values obtained by each of the other observers, since they were derived from twice the number of observations.

The differences between the mean values of the X-gauges derived from the basic étalons L_1 and L_2, and abstracted from the four tables, are :—

X_M in air	$- 0 \cdot 018 \ \lambda_R$
X_M in vacuum	$- 0 \cdot 011$
X_Y in air	$+ 0 \cdot 070$
X_Y in vacuum	$+ 0 \cdot 008$

TABLE XII.—Comparison of Values of X_M in Air obtained from Different Basic Étalons and by Different Observers.

Observer.	Values of X_M in terms of λ_R in Air.			Residual from personal mean.	P.E. of single observation from personal mean.	Residual from grand mean.
	L_1	L_2	Mean.			
J. E. S.	1,553,228·442		1,553,228·501	−0·059	±0·042	−0·160*
	1,553,228·458			−0·043		−0·144†
		1,553,228·527		+0·026		−0·075
		1,553,228·576		+0·075		−0·026
H. B.	1,553,228·636		1,553,228·626	+0·010	±0·049	+0·034
	1,553,228·728			+0·102		+0·126
	1,553,228·586			−0·040		−0·016†
	1,553,228·618			−0·008		+0·016
		1,553,228·654		+0·028		+0·052
		1,553,228·707		+0·081		+0·105
		1,553,228·509		−0·117		−0·093
		1,553,228·566		−0·060		−0·036
R. F. Z.	1,553,228·578		1,553,228·655	−0·077	±0·036	−0·024
	1,553,228·696			+0·041		+0·094†
		1,553,228·663		+0·008		+0·061
		1,553,228·684		+0·029		+0·082
Mean . .	1,553,228·593	1,553,228·611	1,553,228·602 …	P.E. of grand mean = ± 0·015λ_R.		

TABLE XIII.—Comparison of Values of X_M in Vacuum obtained from Different Basic Étalons and by Different Observers.

Observer.	Values of X_M in terms of λ_R in vacuum.			Residual from personal mean.	P.E. of single observation from personal mean.	Residual from grand mean.
	L_1	L_2	Mean.			
J. E. S.	1,552,808·834		1,552,808·930	−0·096	±0·062	−0·083
	1,552,809·054			+0·124		+0·137
		1,552,808·914		−0·016		−0·003
		1,552,808·920		−0·010		+0·003
H. B.	1,552,808·799		1,552,808·897	−0·098	±0·064	−0·118
	1,552,808·863			−0·034		−0·054
	1,552,808·832			−0·065		−0·085
	1,552,808·955			+0·058		+0·038
		1,552,808·898		+0·001		−0·019
		1,552,809·082		+0·185		+0·165*
		1,552,808·940		+0·043		+0·023
		1,552,808·807		−0·090		−0·110
R. F. Z.	1,552,808·913		1,552,808·944	−0·031	±0·046	−0·004
	1,552,809·044			+0·100		+0·127
		1,552,808·893		−0·051		−0·024
		1,552,808·928		−0·016		+0·011
Mean . .	1,552,808·912	1,552,808·923	1,552,808·917	P.E. of grand mean = ± 0·014λ_R.		

* See p. 170.

† See § (*d*), p. 174.

TABLE XIV.—Comparison of Values of X_Y in Air obtained from Different Basic Étalons and by Different Observers.

Observer.	Values of X_Y in terms of λ_R in air.			Residual from personal mean.	P. E. of single observation from personal mean.	Residual from grand mean.
	L_1	L_2	Mean.			
J. E. S.	1,420,227·929		1,420,227·836	+0·093	±0·048	+0·040
	1,420,227·840			+0·004		—0·049
		1,420,227·760		—0·076		—0·129
		1,420,227·815		—0·021		—0·074†
H. B.	1,420,227·951		1,420,227·896	+0·055	±0·033	+0·062
	1,420,227·856			—0·040		—0·033
	1,420,227·951			+0·055		+0·062
	1,420,227·832			—0·064		—0·057
		1,420,227·851		—0·045		—0·038
		1,420,227·932		+0·036		+0·043
		1,420,227·866		—0·030		—0·023†
		1,420,227·928		+0·032		+0·039
R. F. Z.	1,420,227·986		1,420,227·928	+0·058	±0·070	+0·097
	1,420,228·043			+0·115		+0·154*
		1,420,227·818		—0·110		—0·071
		1,420,227·864		—0·064		—0·025
Mean . .	1,420,227·924	1,420,227·854	1,420,227·889	P.E. of grand mean = ±0·012 λ_R.		

* See p. 170. † See § (*d*), p. 174.

TABLE XV.—Comparison of Values of X_Y in Vacuum obtained from Different Basic Étalons and by Different Observers.

Observer.	Values of X_Y in terms of λ_R in vacuum.			Residual from personal mean.	P.E. of single observation from personal mean.	Residual from grand mean.
	L_1	L_2	Mean.			
J. E. S.	1,419,844·129		1,419,844·154	—0·025	±0·034	—0·083
	1,419,844·101			—0·053		—0·111
		1,419,844·170		+0·016		—0·042
		1,419,844·218		+0·064		+0·006
H. B.	1,419,844·240		1,419,844·242	—0·022	±0·038	+0·028
	1,419,844·199			—0·043		—0·013
	1,419,844·339			+0·097		+0·127
	1,419,844·192			—0·050		—0·020
		1,419,844·239		—0·003		+0·027
		1,419,844·169		—0·073		—0·043
		1,419,844·288		+0·046		+0·076
		1,419,844·267		+0·025		+0·055
R. F. Z.	1,419,844·249		1,419,844·209	+0·040	±0·046	+0·037
	1,419,844·277			+0·068		+0·065
		1,419,844·126		—0·083		—0·086
		1,419,844·183		—0·026		—0·029
Mean . .	1,419,844·216	1,419,844·208	1,419,844·212	P.E. of grand mean = ±0·011 λ_R.		

The mean value of the difference for all determinations of X_M and X_Y, taking account of sign, was $+ 0 \cdot 012\ \lambda_R$. It is shown later that the average probable error associated with the mean value of an X-gauge, determined either in air or in vacuum, is of the same order of magnitude as this quantity. This similarity indicates that, even if the red radiation of cadmium is not ideally simple and symmetrical, the effect of any fine structure is too small to be detected by the present methods of measurement applied to the two Fabry-Perot étalons L_1 and L_2, and is therefore negligible for the purpose of these particular determinations.

The differences between personal and grand means taken from the four tables are :—

	J. E. S.	H. B.	R. F. Z.
X_M in air	$- 0 \cdot 101\ \lambda_R$	$+ 0 \cdot 024\ \lambda_R$	$+ 0 \cdot 053\ \lambda_R$
X_M in vacuum	$+ 0 \cdot 013$	$- 0 \cdot 020$	$+ 0 \cdot 027$
X_Y in air	$- 0 \cdot 053$	$+ 0 \cdot 007$	$+ 0 \cdot 039$
X_Y in vacuum	$- 0 \cdot 058$	$+ 0 \cdot 030$	$- 0 \cdot 003$
Average difference . . .	$- 0 \cdot 050\ \lambda_R$	$+ 0 \cdot 010\ \lambda_R$	$+ 0 \cdot 029\ \lambda_R$

These average values of the differences between personal and grand means, which have been calculated taking regard to sign, suggest that quite definite, but small, personal errors are involved in the determinations. The total range of the difference is, however, only about 5 parts in 10^8.

The last three columns in the four tables present the data necessary for the evaluation of the personal probable errors of a single determination and of the probable errors of the grand means for the three observers. The calculations were based on the accepted expressions for probable errors, viz. :—

$$p.e. = 0 \cdot 6745 \sqrt{\frac{\Sigma r^2}{n - 1}},$$

$$\text{P.E.} = 0 \cdot 6745 \sqrt{\frac{\Sigma r^2}{n (n - 1)}},$$

where *p.e.* and P.E. represent respectively the probable errors of a single determination and of the arithmetical mean, Σr^2 is the sum of the squares of the residuals of the individual determinations from the mean, and n is the number of determinations.

The personal probable errors of single determinations collected from the four tables are :—

	J. E. S.	H. B.	R. F. Z.
X_M in air	$\pm 0 \cdot 042\ \lambda_R$	$\pm 0 \cdot 049\ \lambda_R$	$\pm 0 \cdot 036\ \lambda_R$
X_M in vacuum	$\pm 0 \cdot 062$	$\pm 0 \cdot 064$	$\pm 0 \cdot 046$
X_Y in air	$\pm 0 \cdot 048$	$\pm 0 \cdot 033$	$\pm 0 \cdot 070$
X_Y in vacuum	$\pm 0 \cdot 034$	$\pm 0 \cdot 038$	$\pm 0 \cdot 046$
Average value	$\pm 0 \cdot 046\ \lambda_R$	$\pm 0 \cdot 046\ \lambda_R$	$\pm 0 \cdot 050\ \lambda_R$

The average value of the personal probable error of a single determination is nearly the same for each observer, and is of the order of $\pm\ 0\cdot05\ \lambda_R$ or approximately $\pm$ 1 part in 30×10^6 of the lengths of the X-gauges.

The last column of each of the four comparison tables shows the residuals of the individual determinations from the respective grand means. The four values of the probable errors of the grand means are :—

X_M in air	$\pm\ 0\cdot015\ \lambda_R$
X_M in vacuum	$\pm\ 0\cdot014$
X_Y in air	$\pm\ 0\cdot012$
X_Y in vacuum	$\pm\ 0\cdot011$
Average value	$\pm\ 0\cdot013\ \lambda_R$

It will be noted that each probable error of the grand mean has a magnitude slightly less than 1 part in 10^8 of the length of the corresponding X-gauge, despite the presence of the definite personal differences between observers mentioned above. Furthermore, only three determinations, indicated by asterisks in the four tables, of the total of 64 have residuals from the grand means in excess of 10 parts in 10^8 of the lengths of the corresponding X-gauges.

It is of interest to mention that an examination of the measurements of the basic étalons showed that the probable errors of the determinations of L_1 and L_2 were closely one-twelfth and one-ninth, respectively, of the probable errors of the determinations of the X-gauges. The proportionality existing between the probable errors of measurement of the basic étalon and of the X-gauge was referred to in the previous paper (*loc. cit.*, p. 125), where particular examples were given. In the present determinations the probable error of a single determination of either L_1 or L_2 was about $\pm\ 0\cdot005\ \lambda_R$, the magnitude of the probable error for L_1 being about $0\cdot001\ \lambda_R$ less than that for L_2. In a complete determination of an X-gauge, however, a slightly lower probable error was obtained when the basic measurement was made in L_2, owing to the lower factor of multiplication required, which more than compensated for the small initial disadvantage.

The examination of the measurements of L_1 and L_2 also showed that the personal differences between observers, mentioned above in relation to the determinations of X_M and X_Y, appeared in these results in the same sense and in approximately the same proportional magnitude. It is evident, therefore, that the greater part of the experimental error was incurred in the basic measurements of length in L_1 and L_2, and that the increase of proportionate error due to the subsequent procedure of optical multiplication and the measurement of $(L_6 - X)$ was exceedingly small. The application of photography to the recording of the interference rings produced by the basic étalons, and the subsequent measurement of the excess fractions from the photographic records by means of a microphotometer, would possibly eliminate the personal experimental errors and so lead to a notable enhancement of the present accuracy of measurement.

(*b*) *Determinations of* $(M' - X_M)$ *and* $(Y' - X_Y)$.—Table XVI shows for comparison the values of $(M' - X_M)$ and $(Y' - X_Y)$ obtained from the two auxiliary blocks by the three observers. The values of $(M' - X_M)$ and $(Y' - X_Y)$ are the corrected values of these quantities taken from Tables VIII and XI respectively. Mechanical measurements of the two blocks indicated that block (3, 4) was larger than block (1, 2) by about 1×10^{-6} inch or 0·04 λ_R. From Table XVI the difference between block (3, 4) and block (1, 2) was + 0·003 λ_R for determinations of $(M' - X_M)$ and + 0·086 λ_R for determinations of $(Y' - X_Y)$, giving a mean value of about + 0·04 λ_R.

TABLE XVI.—Comparison of Values of $(M' - X_M)$ and $(Y' - X_Y)$ obtained from Blocks (1, 2) and (3, 4) by Different Observers.

Observer.	Values of $(M' - X_M)$.			Residuals.		Values of $(Y' - X_Y)$.			Residuals.	
	Block (1, 2) (λ_R).	Block (3, 4) (λ_R).	Mean (λ_R).	From mean for (1, 2).	From mean for (3, 4).	Block (1, 2) (λ_R).	Block (3, 4) (λ_R).	Mean (λ_R).	From mean for (1, 2).	From mean for (3, 4).
J. E. S.	9·286 9·266	9·253 9·265	9·268	+0·021 +0·001	−0·015 −0·003	44·008 44·046	44·095 44·113	44·065	+0·018 +0·056	+0·019 +0·037
H. B.	9·351 9·267 9·277 9·165	9·320 9·268 9·275 9·226	9·268	+0·086 +0·002 +0·012 −0·100	+0·052 0 +0·007 −0·042	44·081 44·047 43·890 44·050	44·120 43·998 44·104 44·115	44·051	+0·091 +0·057 −0·100 +0·060	+0·044 −0·078 +0·028 +0·039
R. F. Z.	9·306 9·200	9·300 9·238	9·261	+0·041 −0·065	+0·032 −0·030	43·902 43·893	43·973 44·094	43·966	−0·088 −0·097	−0·103 +0·018
Mean .	9·265	9·268	9·266	P.E. of grand mean = $\pm 0{\cdot}008\lambda_R$		43·990	44·076	44·033	P.E. of grand mean = $\pm 0{\cdot}011\lambda_R$.	

Personal differences between the determinations of $(M' - X_M)$ were practically negligible. In the determinations of $(Y' - X_Y)$ one observer obtained a low mean value as compared with the means obtained by the other two observers. Some part of this difference may have been due to the influence of external temperature conditions, for this observer made three of his comparisons before 8 a.m., when room temperatures were falling, whereas all other comparisons were made after 9 p.m., when room temperatures were generally rising.

Since it was evident that some small difference actually existed between the sizes of of the blocks the residuals shown in Table XVI were calculated with due regard to the particular blocks from which the individual values of $(M' - X_M)$ and $(Y' - X_Y)$ were obtained. The 16 residuals thus derived from each of the two series of comparisons were then used to calculate the probable errors of the mean values of $(M' - X_M)$ and $(Y' - X_Y)$ in the usual manner, the probable error of the mean for the determination of $(M' - X_M)$ being $\pm 0 \cdot 008\ \lambda_R$ and for the determination of $(Y' - X_Y)$ being $\pm 0 \cdot 011\ \lambda_R$.

It is of interest at this stage to calculate the combined probable error associated with the mean values of the composite gauges in terms of wave-lengths. In this calculation account has to be taken of the probable errors associated with :—

(*a*) the mean values of the X-gauges,
(*b*) the mean values of the fixed corrections enumerated in sections 4 (*b*) and 5 (*b*),
(*c*) the mean values of $(M' - X_M)$ and $(Y' - X_Y)$.

Since the total fixed correction for determinations in vacuum includes two items in addition to those which have to be considered for the determinations in air, the calculation of the combined probable error for each condition is made independently.

The probable errors of the mean values of the fixed corrections were evaluated by the usual method in the following manner. In the correction to the axial length of L_6 the four mean values of the correction, both in air and in vacuum, determined from the four wringings of the quartz plates (see Table VI), were treated as independent observations and the residuals of these from the grand means were used in the calculation of the required probable error. The probable error associated with the correction to the mechanical lengths of the X-gauges was obtained from the information given by ROLT and BARRELL (*loc. cit.*), in which the error is stated in terms of the root mean square residual ; from this the probable error according to the usual formula has been calculated. For the " reflection " correction the five values given in Table VII were treated as five independent observations and the residuals of these from the mean value were used to calculate the required probable error. The probable error corresponding to the correction for the elastic expansion of the steel X-gauges in vacuum was obtained by calculating the effect of the variations given on p. 155, with the mean values of the elastic constants, on the values of the correction actually applied. Finally, the probable error associated with the residual pressure correction was obtained by calculating the effect of the variations of residual pressure, between 0·017 mm. and 0·023 mm., on the flat correction for a residual pressure of 0·020 mm. which was actually applied.

Thus, for determinations in air the individual probable errors in terms of λ_R were :—

$$(a)\quad \left.\begin{array}{l} \pm 0\cdot015 \text{ for } X_M \\ \pm 0\cdot012 \text{ for } X_Y \end{array}\right\} \text{Average value} \;\ldots\ldots\ldots\ldots\quad \pm 0\cdot014$$

(*b*)	For correction to axial length of L_6	± 0·004
	For correction to mechanical lengths	± 0·006
	For "reflection" correction	± 0·003
(*c*)	$\left.\begin{array}{l}\pm 0{\cdot}008 \text{ for } (M' - X_M)\\ \pm 0{\cdot}011 \text{ for } (Y' - X_Y)\end{array}\right\}$ Average value	± 0·010

The combined probable error of the mean values of the composite gauges for determinations in air is therefore ± 0·019 λ_R, as determined by applying the usual rule for the combination of errors to the individual values given above.

For determinations in vacuum, the individual probable errors in terms of λ_R were :—

(*a*)	$\left.\begin{array}{l}\pm 0{\cdot}014 \text{ for } X_M\\ \pm 0{\cdot}011 \text{ for } X_Y\end{array}\right\}$ Average value	± 0·013
(*b*)	For correction to axial length of L_6	± 0·003
	For correction to mechanical lengths	± 0·006
	For "reflection" correction	± 0·003
	For correction to 1 atmosphere	± 0·007
	For residual pressure correction	± 0·002
(*c*)	$\left.\begin{array}{l}\pm 0{\cdot}008 \text{ for } (M' - X_M)\\ \pm 0{\cdot}011 \text{ for } (Y' - X_Y)\end{array}\right\}$ Average value	± 0·010

The combined probable error of the mean values of the composite gauges for determinations in vacuum is therefore ± 0·019 λ_R.

If account is further taken of the possibility that errors of temperature measurement may amount to ± 0·001° C., corresponding to an error in the lengths of the steel gauges of ± 0·017 λ_R, then the overall probable error associated with the optical measurements becomes ± 0·025 λ_R, which is less than 2 parts in 10^8 of the lengths concerned.

(*c*) *Comparisons of the Composite Gauges with the Fundamental Standards of Length.*—The experimental error associated with the optical stages of the complete determination of the fundamental standards of length in terms of wave-lengths of light is practically negligible by comparison with the experimental error of the purely metrological operations involved in the final stage. In the authors' opinion the average accuracy of repetition generally attainable in the comparison of two line-standards of the best class, such, for example, as the Prototype Metre and its National Copies, is of the order of $0{\cdot}25 \times 10^{-6}$ M, which corresponds to a possible range of values of about ± 1 part in 8,000,000. Differences of this order of magnitude are liable to occur between the results of comparisons of the same bars by different observers. Line-standard comparisons involving the use of the Imperial Standard Yard are liable to show even greater variations owing to the inferior definition afforded by its terminal lines.

The comparison of the composite gauge M′ with the Prototype Metre involves effectively either three of four stages. M′ was first compared with the British National Copy of the Metre, known as P.I. 16, which was recently compared with the working standards of the Bureau International. The working standards of the Bureau are normally compared with the " Témoins " of the International Metre, which are in turn compared with the Prototype Metre itself. The working standards have, however, on at least two occasions been compared directly with the Prototype. The comparison of the composite gauge Y′ with the Imperial Standard Yard was performed in one stage, and therefore the final accuracy of the yard determinations is probably of the same order as that obtained in the metre determinations in view of the increased number of comparisons involved in the latter.

(*d*) *Determination of Refractive Index of Air.*—Since the determination of refractive index of air is incidental to the main determinations of length and is, in fact, resolved into a calculation of the ratio of the length of an X-gauge in terms of wave-lengths in air to its length in terms of wave-lengths in vacuum, an estimate of the probable error of the value found for the refractive index can be made directly from the information already given. It should be remembered that the refractive index was calculated after the application of the fixed corrections to the lengths of the X-gauges ; of these fixed corrections two are common to the determinations both in air and in vacuum, viz., the correction from optical to mechanical length and the " reflection " correction, and the probable errors of the mean values of these corrections may therefore be omitted from the calculations. The individual probable errors concerned in the calculations of refractive index were :—

Average for mean values of the X-gauges in air	$\pm 0 \cdot 014\ \lambda_R$
Mean correction to axial length of L_6 in air	$\pm 0 \cdot 004$
Average for mean values of the X-gauges in vacuum	$\pm 0 \cdot 013$
Mean correction to axial length of L_6 in vacuum	$\pm 0 \cdot 003$
Mean correction to 1 atmosphere	$\pm 0 \cdot 007$
Mean correction for residual pressure	$\pm 0 \cdot 002$

The combined probable error due to these individual errors is $\pm 0 \cdot 021\ \lambda_R$; the average length of the X-gauges is $1 \cdot 49 \times 10^6\ \lambda_R$ and therefore the proportional error is $\pm 1 \cdot 4$ parts in 10^8, so that the estimated probable error of the mean value of refractive index is $\pm 0 \cdot 000,000,014$. The mean values of $(\mu_0 - 1)$ obtained from determinations of X_M and X_Y were $291 \cdot 841 \times 10^{-6}$ and $291 \cdot 861 \times 10^{-6}$ respectively, so that the actual difference was well within the estimated range of the probable error.

Reference has already been made to the fact that the determinations in air were generally accomplished with different samples of air. On two occasions, however, observations were made by different observers on the same sample of air. Thus the determinations XVII, XIX, and XXI of X_M, indicated by " daggers " in Table XII, were made in this manner, and it will be seen that they happen to cover a range very similar to that over which the rest of the determinations extend, and that the personal differences exhibit the characteristic sense of the whole series. Likewise the determinations XVII and XIX of X_Y, similarly indicated in Table XIV, were made by two observers in the same sample of air, and here the personal difference is again characteristic. It may therefore be generally stated that observations in different samples of air display no definitely greater variations than those made in the same sample. Since the probable errors of the mean results for X_M and X_Y, in air and in vacuum, were alike and of the order $\pm$ 1 part in 10^8 of the length concerned, then the mean values of $(\mu_0 - 1)$ for the two series of samples of air in which the X-gauges were determined can be considered as established to an order of accuracy of about $\pm$ 1 part in 30,000, as the actual value of $(\mu_0 - 1)$ is approximately 0·0003. Assuming that the density of air varies linearly with $(\mu_0 - 1)$, then the mean density of air for each of the two series of samples was equal to within $\pm$ 1 part in 30,000.

Similarly, as the probable errors of single determinations of X_M and X_Y, in air and in vacuum, were practically identical and of the order of $\pm$ 1 part in 30×10^6 of the length concerned, the daily values of $(\mu_0 - 1)$ can be regarded as constant to an order of accuracy of at least $\pm$ 1 part in 10,000. Making the same assumption with regard to density, then it may be stated that the density, under standard conditions, of 29 different samples of air taken over a period of about two months was constant to at least $\pm$ 1 part in 10,000. This conclusion is of interest because direct measurements of the density of air under standard conditions have appeared to exhibit larger variations than this, amounting in the recent determinations of STOCK, RAMSER, and EYBER* to $\pm$ 4 parts in 10,000.

6. *Final Adjusted Results.*

It was decided to adopt the mean of the two values of $(\mu_0 - 1)$, calculated from the determinations of the metre and yard X-gauges, X_M and X_Y, in air and in vacuum, as a basis for the final adjustment of all the results. The values of X_M and X_Y in terms of λ_R in air and in vacuum, given in sections 3 (*c*) and 4 (*c*), were first suitably adjusted by small amounts depending on the difference between the two values of $(\mu_0 - 1)$, so that in the end the adjusted pairs of values for X_M and X_Y gave closely the same result for $(\mu_0 - 1)$. Then, confining attention to the measurements in terms of λ_R in vacuum, the final values of M′ and Y′ were calculated from the adjusted values of X_M and X_Y by use of the differences $(M' - X_M)$ and $(Y' - X_Y)$ given in sections 3 (*d*) and 4 (*d*), from which the values of M and Y were subsequently derived by use of the relation between M′ and M and between Y′ and Y given in sections 3 (*e*) and 4 (*e*). Having thus obtained the values of M and Y in terms of λ_R in vacuum, their values in terms of λ_R in air under various conditions were calculated, using values of the refractive index of air under these conditions derived from the mean value of $(\mu_0 - 1)$.

---

* ' Z. phys. Chem.,' A, vol. 163, p. 82 (1933).

The calculations of the final adjusted results are set out below :—

Value of $(\mu_0 - 1)$ from determinations of X_M . . . . $= 291 \cdot 841 \times 10^{-6}$
Value of $(\mu_0 - 1)$ from determinations of X_Y . . . . $= 291 \cdot 861 \times 10^{-6}$

Mean value of $(\mu_0 - 1)$ . . . . $= 291 \cdot 851 \times 10^{-6}$
Difference between values of $(\mu_0 - 1)$ . . . . $= 0 \cdot 020 \times 10^{-6}$

Since the first value of $(\mu_0 - 1)$ was derived from two determinations of X_M, one in terms of λ_R in air and the other in terms of λ_R in vacuum, and the second value from two similar determinations of X_Y, therefore each one of the four values of the X-gauges was adjusted by an amount equal to $0 \cdot 005 \times 10^{-6}$ of its magnitude. The signs of the four adjustment corrections thus obtained were arranged so that the value of $(\mu_0 - 1)$ from X_M was increased while that from X_Y was decreased. The following calculations give the final values of the metre and yard X-gauges, X_M and X_Y respectively, at the mean temperatures of observation, in terms of λ_R in air and in vacuum. The values in air refer throughout to air at 760 mm. pressure, containing no moisture nor carbon dioxide :—

X_M at 21·005° C. . . . . = 1,553,228·926 λ_R in air at 21·005° C.
Adjustment . . . . = + 0·008

Adjusted value of X_M at 21·005° C. . = 1,553,228·934 λ_R (21·005° C.).

X_M at 21·005° C. . . . . = 1,552,808·563 λ_R in vacuum.
Adjustment . . . . = − 0·008

Adjusted value of X_M at 21·005° C. . = 1,552,808·555 λ_R (vac.).

X_Y at $21 \cdot 351_5$° C. . . . . = 1,420,228·213 λ_R in air at $21 \cdot 351_5$° C.
Adjustment . . . . = − 0·007

Adjusted value of X_Y at $21 \cdot 351_5$° C. . = 1,420,228·206 λ_R $(21 \cdot 351_5$° C.).

X_Y at $21 \cdot 351_5$° C. . . . . = 1,419,844·279 λ_R in vacuum.
Adjustment . . . . = + 0·007

Adjusted value of X_Y at $21 \cdot 351_5$° C. . = 1,419,844·286 λ_R (vac.).

Also since the mean value of $(\mu_0 - 1) = 291 \cdot 851 \times 10^{-6}$ and α is assumed to have PÉRARD'S value of 0·003716 :—

$$(\mu_{15} - 1) = \frac{291 \cdot 851 \times 10^{-6}}{1 + 15\alpha} = 276 \cdot 442 \times 10^{-6},$$

$$(\mu_{20} - 1) = \frac{291 \cdot 851 \times 10^{-6}}{1 + 20\alpha} = 271 \cdot 661 \times 10^{-6}.$$

If λ_R be the wave-length of the cadmium red radiation in vacuum :—

$$X_M \text{ at } 21\cdot005^\circ \text{ C.} = 1{,}552{,}808\cdot555\ \lambda_R$$

$$(M' - X_M) \quad ,, \quad ,, \quad = \quad 9\cdot264$$

Therefore $$M' \quad ,, \quad ,, \quad = 1{,}552{,}817\cdot819$$

But $$M' \text{ at } 21\cdot005^\circ \text{ C.} = 1\cdot000{,}053{,}65\ M.$$

Therefore

$$M = \frac{1{,}552{,}817\cdot819}{1\cdot000{,}053{,}65}\lambda_R = \underline{\underline{1{,}552{,}734\cdot515\ \lambda_R\ (\text{vac.})}}.$$

$$X_Y \text{ at } 21\cdot351_5^\circ \text{ C.} = 1{,}419{,}844\cdot286\ \lambda_R$$

$$(Y' - X_Y) \quad ,, \quad ,, \quad = \quad 44\cdot021$$

Therefore $$Y' \quad ,, \quad ,, \quad = 1{,}419{,}888\cdot307.$$

But $$Y' \text{ at } 21\cdot351_5^\circ \text{ C.} = 1\cdot000{,}049{,}30\ Y.$$

Therefore

$$Y = \frac{1{,}419{,}888\cdot307}{1\cdot000{,}049{,}30}\lambda_R = \underline{\underline{1{,}419{,}818\cdot310\ \lambda_R\ (\text{vac.})}}.$$

If λ_R be the wave-length of the cadmium red radiation in air at 15° C., where $\mu_{15} = 1\cdot000{,}276{,}442$, then :—

$$M = \mu_{15} \times 1\cdot552{,}734\cdot515\ \lambda_R = \underline{\underline{1{,}553{,}163\cdot756\ \lambda_R\ (15^\circ \text{ C.})}}$$

and

$$Y = \mu_{15} \times 1{,}419{,}818\cdot310\ \lambda_R = \underline{\underline{1{,}420{,}210\cdot807\ \lambda_R\ (15^\circ \text{ C.})}}.$$

If λ_R be the wave-length of the cadmium red radiation in air at 20° C., where $\mu_{20} = 1\cdot000{,}271{,}661$, then :—

$$M = \mu_{20} \times 1{,}552{,}734\cdot515\ \lambda_R = \underline{\underline{1{,}553{,}156\cdot332\ \lambda_R\ (20^\circ \text{ C.})}}$$

and

$$Y = \mu_{20} \times 1{,}419{,}818\cdot310\ \lambda_R = \underline{\underline{1{,}420{,}204\cdot019\ \lambda_R\ (20^\circ \text{ C.})}}.$$

The corresponding values of λ_R in terms of the Prototype Metre are :—

In vacuum	$6440\cdot2509_9 \times 10^{-10}$ M
In air at 15° C.	$6438\cdot4711_3$
In air at 20° C.	$6438\cdot5019_0$

The final results of the determinations of the metre and the yard are collected together in Table XVII, in which the individual values are rounded off to an accuracy more compatible with the total experimental accuracy of the optical determinations. But if the accuracy attainable in the purely metrological stages is taken into account the values of the wave-numbers can only be regarded as established to a final accuracy of the order of about $\pm 0\cdot25\ \lambda_R$, corresponding to an accuracy in the values of the wave-lengths of $\pm 0\cdot001 \times 10^{-10}$ M. The values of the refractive index of air for the red radiation of cadmium depend to a certain extent on the accuracy of PÉRARD's value of α, but apart from this it is estimated that the values given in the table are probably correct to within 1 or 2 units of the last figure quoted.

TABLE XVII.—Final Results of the Determinations of the Metre and the Yard.

Condition.	Wave-numbers		Wave-length (1×10^{-10} metre).	Refractive index ($\mu_0 = 1\cdot000,291,85$).
	Metre.	Yard.		
Vacuum . . .	1,552,734·52	1,419,818·31	6440·2510	—
Air at 15° C. . .	1,553,163·76	1,420,210·81	6438·4711	1·000,276,44
Air at 20° C. . .	1,553,156·33	1,420,204·02	6438·5019	1·000,271,66

The wave-length in air at 15° C. is compared in Table XVIII with the results obtained by previous observers.

TABLE XVIII.—Comparison of Values of λ_R (15° C.) Obtained by Different Observers.

Date.	Observers.	λ_R (10^{-10} M).		Difference from mean (10^{-10} M).
		As originally given.	After adjustment to uniform conditions.	
1895	MICHELSON and BENOÎT	6438·4722*	6438·4691	—0·0005
1905–6	BENOÎT, FABRY, and PEROT . . .	6438·4696†	6438·4703	+0·0007
1927	WATANABE and IMAIZUMI	6438·4685‡	6438·4682	—0·0014
1933	SEARS and BARRELL	6438·4711	6438·4708	+0·0012
Mean			6438·4696	

* 'Trav. Bur. int. Pds. Mes.,' vol. 11, p. 85 (1895).
† 'Trav. Bur. int. Pds. Mes.,' vol. 15, p. 131 (1913).
‡ 'Proc. Imp. Acad. Tokyo,' vol. 4, p. 351 (1928).

The results in the fourth column of Table XVIII have been adjusted, as far as practicable, to a uniform "standard" condition corresponding to measurements made in dry air containing a normal proportion of carbon dioxide at a temperature of 15° C. on the normal hydrogen scale.

The determinations of MICHELSON and BENOÎT were made in ordinary atmospheric air, and their original result has been adjusted to the dry air condition on the assumption of an average 50% humidity. It has also been corrected to take account of subsequent changes in the accepted values of the coefficients of expansion of the metre bars of the Bureau International which were used as the basis of reference. The original result of the BENOÎT-FABRY-PEROT determination was given in terms of dry air, but again has been corrected for changes in the accepted equations of the reference standards employed. The adjusted values for these two determinations are those given by GUILLAUME.* The amount of carbon dioxide present in these cases is unknown and can only be assumed as approximately equal to the normal proportion of 0·03%.

The original result of WATANABE and IMAIZUMI was given for dry air containing no carbon dioxide and has been adjusted by $-0\cdot0003 \times 10^{-10}$ M, corresponding to the introduction of the normal proportion of 0·03% of carbon dioxide. The same adjustment has been made to the present result. In addition, a further small adjustment is theoretically necessary on account of the fact that all temperature measurements in the present investigation are in the International Temperature Scale, whereas the earlier results refer to the normal hydrogen scale. According to HALL† the indication of the hydrogen thermometer at 15° C., taking the average value determined by different observers, is approximately 0·001° C. in excess of the temperature on the thermodynamic scale. This corresponds to a change of $-0\cdot009 \times 10^{-6}$ M in the basis of measurement due to the thermal expansion of the platinum-iridium metre, or to $-0\cdot0000_6 \times 10^{-10}$ M on the wave-length, the change in the refractive index of air for 0·001° C. being entirely negligible. Having regard to the rounding off from the subsequent decimal place it happens that this adjustment is too small to affect the figure given in Table XVIII.

It may be observed in the first place that of four completely independent determinations the maximum divergence from the mean $(0\cdot0014 \times 10^{-10}$ M) amounts only to 2·2 parts in 10^7, which is quite within the possible limits of variation of the different line standard comparisons on which they are based. It is also interesting to note that the mean value is by chance identical with the value originally given by BENOÎT, FABRY, and PEROT, which has since received international sanction in the definition of the Angstrom unit.

It will be noticed that the new value for the wave-length in vacuum agrees within $0\cdot0002 \times 10^{-10}$ M with that quoted in the paper (*loc. cit.*, p. 125), giving the results

---

* "La Création du Bureau International," Paris, Gauthier-Villars (1927).

† 'Phil. Trans.,' A, vol. 229, p. 45 (1930).

of the preliminary measurements made with the apparatus with which these determinations have been carried out. The value in air at 15° C. shows a difference of $0\cdot0006 \times 10^{-10}$ M. This is presumably due to the fact that the condition of the air was not precisely ascertained in the preliminary experiments. It must be remembered that the preliminary and final measurements depend on two entirely independent line standard determinations, the former being based on a comparison of the N.P.L. nickel bar No. 184 with the working standards of the Bureau in 1922, and the latter on a comparison with the British National Copy of the Metre No. 16.

Had the results of the present determinations been calculated on the basis of the 1933 comparison of the British Metre No. 16 at the Bureau International (see Appendix III, p. 213) the value in "standard" air would have been reduced to $6438\cdot4700 \times 10^{-10}$ M, which is still nearer to the Benoît-Fabry-Perot result, and also to the mean result of all observers.

The refractive index for the red radiation of cadmium derived from the provisional measurements was 1·000,276,45, for air at 15° C. containing an assumed normal amount of carbon dioxide. When corrected for the assumed carbon dioxide content of 0·03%, the refractive index becomes 1·000,276,40, as compared with the value 1·000,276,44 resulting from the present determinations. The refractive index calculated from Pérard's data (*loc. cit.*) is 1·000,276,37 and from the data of Meggers and Peters (*loc. cit.*) 1·000,275,77, both values applying to dry air, at 15° C. and 760 mm., containing no carbon dioxide.

7. *The Ratio of the Yard to the Metre.*

Incidentally, the results of the present investigation afford a new value for the ratio of the yard to the metre, namely :—

$$\frac{\text{Yard}}{\text{Metre}} = \frac{1{,}419{,}818\cdot31}{1{,}552{,}734\cdot52} = 0\cdot914{,}398{,}62.$$

This figure compares well with that recently determined at the National Physical Laboratory from metrological measurements, which was 0·914,398,41 (Sears, Johnson, and Jolly, *loc. cit.*). In making this comparison it must be remembered that the value now found depends on entirely fresh series of metrological comparisons both with the Imperial Standard Yard and with the International Metre. The concordance actually found is better than might be expected under these conditions, and must, in fact, be regarded as to some extent fortuitous. The metrological comparisons referred to include :—

(*a*) the determination of the yard and metre lengths on the Laboratory nickel reference bar No. 184 ;

(*b*) the comparison of these lengths with those of the composite yard and metre gauges.

If the results of the comparisons included under (*a*) are combined with the known ratios of the yard and metre lengths on No. 184, as in the previous paper, the value found for the ratio is 0·914,398,26.

Further consideration of these results is deferred for inclusion in a separate paper which is being prepared, dealing with the history of the relationship of the Yard to the Metre.

8. *Notes on the Possible Establishment of a Wave-length Unit of Length.*

In the introduction to the previous paper a brief historical survey was given of the circumstances which led to the present investigation being undertaken, and which may be expected to lead eventually to the establishment of a wave-length of light as the ultimate basis of definition for the fundamental units of length. In the following notes three important aspects of this problem are discussed, namely, the selection of the monochromatic radiation to serve as the basis of reference, the influence of the refractive index of air upon measurements of length in terms of wave-lengths of light, and the definition of the existing units of length in terms of the wave-length of the selected radiation. As a consequence of the work described in this paper certain tentative proposals for the possible establishment of wave-length units of length are outlined.

At the present time the wave-length of the red radiation of cadmium in "normal" air—viz., dry air at 15° C., under a pressure of 760 mm. of mercury at 0° C. ($g = 980 \cdot 665$ cm. per sec. per sec.) and containing 0·03% by volume of carbon dioxide—as determined by BENOÎT, FABRY, and PEROT,* is recognized in spectroscopy as the reference standard for the measurement of all other wave-lengths. It has also been given provisional sanction for use in metrology as an alternative to direct comparison with the international metre.† The results of the successive determinations made by MICHELSON and BENOÎT, by BENOÎT, FABRY, and PEROT, by WATANABE and IMAIZUMI, and by the present authors being in agreement within limits of variation which may be attributed to the use of different copies of the metre, it follows that any convenient value within this range might safely be adopted as a basis for the future definition of the metre without measurable change in the value of the unit as defined by the present standards. Clearly, in these circumstances, the wave-length of the radiation adopted

---

* No account was taken in the BENOÎT-FABRY-PEROT determination of the influence of carbon dioxide, normally present in the atmosphere, upon the refractive index of air and consequently upon the value of the determined wave-length. It is therefore usually assumed that the amount of carbon dioxide in the air surrounding the original apparatus had the normal proportion of 0·03% generally found in fresh air. At ordinary temperatures the refractive index, for the cadmium red radiation, of air containing 0·03% of carbon dioxide is greater than that of air containing no carbon dioxide by 0·000,000,045, as determined by calculation from the data given by PÉRARD (*loc. cit.*, pp. 78, 82).

† 'Proc.-verb. Com. int. Poids Mes.,' p. 67 (1927).

for the future definition of the unit of length should preferably be so chosen as to preserve the present accepted value of the wave-length of the red radiation of cadmium in "normal" air, namely, $6438 \cdot 4696 \times 10^{-10}$ metre, which incidentally agrees (see Table XVIII) with the mean of the four determinations so far published, and therefore is certainly close to the true value.

Whether the cadmium red is definitely the best radiation that might be chosen as the basis for the future definitions of the units of length has not, perhaps, been finally established. There are reasons, based partly on theoretical considerations and partly on evidence obtained with interferometers of very high resolving power, for suspecting that this radiation has an associated hyperfine structure. At the same time the agreement obtained in the present investigation between measurements of length based on observations in two Fabry-Perot étalons of different lengths indicates that such hyperfine structure and/or asymmetry as may be present in the radiation have no appreciable effect on the results of practical measurements of length of the type described. This conclusion is in some measure confirmed by the agreement* between the values of the metre, in terms of wave-lengths of the cadmium red radiation in air, obtained by Benoît, Fabry, and Perot, and by the present authors, for the two results were based effectively on observations on étalons of three different lengths. While certain other radiations, notably of krypton, have been suggested as possible alternatives to the red radiation of cadmium, it cannot be said, in the present state of our knowledge, that these have been definitely proved superior to it in possessing all the attributes desirable in the radiation selected to serve as the ultimate basis of reference for all measurements of length. The question is one undoubtedly deserving of further study, but in the meantime there does not appear to be sufficient justification for superseding the cadmium red radiation as a basis of reference.

The standard sources for the cadmium red radiation hitherto officially recognized in spectroscopy and metrology are effectively copies of the discharge tube originally used by Michelson. In comparison with modern sources of monochromatic radiations the Michelson lamp is definitely unsuitable as a source of the standard radiation. Thus, for instance, the lamp requires comparatively high temperature and voltage for its satisfactory excitation, both of which requirements are inconvenient in use; it normally has a short life and the red radiation it produces is of low intensity.

On the other hand, the hot-cathode lamp of the type employed in the present investigation does not suffer from these disadvantages, except possibly from that due to the moderately high temperature automatically developed by the exciting current. Furthermore, it is known that the wave-length of the red radiation produced from either the Michelson or Osram lamp is identical within the limits of experimental error normally associated with interferential comparisons of two similar radiations.

---

* It has already been mentioned that this agreement must be regarded as to some extent fortuitous, on account of the experimental errors normally associated with comparisons of line-standards.

Another alternative source for the red radiation of cadmium is the Schüler hollow-cathode lamp, which is being increasingly used, in modern investigations of hyperfine structure, for producing spectrum lines of extreme sharpness and of very high intensity. It is believed that no comparisons of this type of lamp with the Michelson and Osram lamps have yet been made, and the authors have such comparisons in contemplation.

In the present state of knowledge it would appear that no special restrictions need to be imposed regarding the type of cadmium lamp employed as a source of the standard radiation so long as care is taken to ensure that the frequency of the radiation is not affected by perturbations due to the conditions of excitation.

The next question is concerned with the conditions under which the wave-length of the chosen radiation is to be regarded as standard. At the present time the wave-length in " normal " air, the conditions of which are controlled by references to temperature and pressure, and also to composition in so far as it is affected by water vapour and carbon dioxide, is taken as standard. Obviously, however, the primary definition would be much simplified if the wave-length in vacuum were adopted for this purpose.

The present investigation shows little difference in the final accuracy of practical measurements of length, whether they are made in terms of waves in air or in terms of waves in vacuum, provided that the conditions of the air are adequately controlled and measured. Nevertheless, the advantage appears to lie with a definition in vacuum mainly because the wave-length is then a truly natural constant which is quite easily reproducible under modern conditions of vacuum technique. In spectroscopic practice also the wave-length in vacuum is a more fundamental unit, particularly in the analysis of spectra, for which purpose the positions of lines are usually specified by means of wave-numbers, *i.e.*, the numbers of waves per centimetre in vacuum.

It may perhaps be argued that, just as with the present definition of the metre as the length of a certain bar at 0° C., comparisons cannot be conveniently and consequently are not normally made at 0° C., with the result that difficulties continually arise from the necessity of applying corrections for the thermal expansions of the bars involved in any comparisons, so with a definition in terms of wave-lengths in vacuum it will always be necessary to apply corrections when measurements are made in air, as in practice they are likely for the most part to be made. There is, however, this fundamental difference that, whereas in the case of the bars every bar has its own different coefficient of expansion which must be determined before accurate results can be obtained, the necessary data for correcting wave-length measurements from air to vacuum, or *vice-versa*, can be established once and for all by suitable determinations of the refractive index of air under different conditions and for different wave-lengths. The values so determined can receive official sanction and be generally applied until eventually superseded as the result of new determinations of superior precision. The results of all measurements made in air must be reduced to some standard condition, and the differences between the results of such measurements, when reduced respectively to " standard " air or to vacuum, will be constant and correct to the accuracy to which the refractive index of "standard" air is known at the time when they are made.

An important advantage of the definition in vacuum arises from the fact that it is possible to make, if desired, primary determinations of lengths and wave-numbers by direct measurement in vacuum, thereby avoiding all complications associated with the refraction and dispersion of air.

It is considered, therefore, that the weight of ultimate advantage is greatly in favour of adopting a wave-length in vacuum as the basis of definition for the units of length.

It will be understood from the foregoing that the establishment of a wave-length definition for the unit of length, whether in air or in vacuum, requires for its satisfactory completion new and more precise determinations of the refraction and dispersion of air, including also a complete examination of the influence of the variable atmospheric factors, viz., temperature, pressure, humidity, and carbon dioxide content, on these physical properties. The present investigation has incidentally supplied some limited information regarding the refractive index of air for the red radiation of cadmium. It is intended shortly to make more complete determinations of the refraction and dispersion of air by a modification of the existing apparatus, which will enable the measurements to be made directly and not, as at present, through the agency of the X-gauges. At first these determinations will only be made in the visible spectrum, thereby providing all the necessary data for metrological work, which chiefly employs visible light for its operations; then a further simple modification of the apparatus would make it possible to use photographic means for extending the determinations into the infra-red and ultra-violet regions, thereby obtaining the additional information required for spectroscopic purposes.

It will obviously be preferable to defer any final proposals concerning the future definition of the units of length in terms of wave-lengths of light until the results of these further investigations have been obtained. Assuming, however, for the moment that the value found for the refractive index of air in the course of the present work is reasonably correct, it is possible to follow a little further the results of applying the principles enunciated to a definition of the metre and the yard in terms of the cadmium red radiation in vacuum.

The present accepted value of the relationship between the metre and the wave-length of the cadmium red radiation is that determined by BENOÎT, FABRY, and PEROT, and is:—

$$1 \text{ metre} = 1{,}553{,}164 \cdot 13\ \lambda_R \text{ in "normal" air,}$$

$$\text{or } \lambda_R \text{ ("normal" air)} = 6438 \cdot 4696 \times 10^{-10} \text{ metre.}$$

The value of the refractive index of air at 15° C. and 760 mm. pressure, containing no water vapour nor carbon dioxide, is $1 \cdot 000{,}276{,}44_2$ as determined by the present investigation. If 0·03% of the air is assumed to be replaced by carbon dioxide the refractive index is increased by 0·000,000,045 (see footnote, p. 181), so that the refractive index of " normal " air becomes, after rounding off to an appropriate accuracy, 1·000,276,49. Using this information, the accepted relationship is now converted into the terms of the wave-length in vacuum thus :—

$$1 \text{ metre} = \frac{1{,}553{,}164\cdot 13}{1\cdot 000{,}276{,}49} = 1{,}552{,}734\cdot 81\ \lambda_{\mathrm{R}} \text{ in vacuum,}$$

$$\text{or } \lambda_{\mathrm{R}} \text{ (vacuum)} = 6440\cdot 2498 \times 10^{-10} \text{ metre.}$$

Referring to Table XVII, it will be seen that the value obtained by direct measurement of the metre is 1,552,734·52 λ_{R}, which differs by less than 2 parts in 10^7 from the above value. The difference is within the uncertainty of definition of existing metre standards.

Since the present definition of the International Angstrom unit is contained in the statement that the wave-length of the cadmium red line in " normal " air is 6438·4696 A., and the suggested definition of the metre is based on the same value reduced to vacuum by the aid of a correction derived from a precise determination of the refractive index of air, the value of the Angstrom would be automatically preserved by re-defining it for the future simply as 10^{-10} M.

With regard to the yard, it is suggested that advantage might be taken of the opportunity afforded by its re-definition in terms of wave-lengths of light to choose the definition in such a manner that the simple approximate ratio of 1 yard = 0·9144 metre (or 1 inch = 25·4 mm.) should for the future be accurately true. This simple factor differs by less than 2 parts in 10^6 from the true value at the present time (*vide* section 7). It is closely the mean between the yards at present current in Great Britain and the United States, and is, in fact, already generally employed in both countries for industrial purposes. Its adoption, moreover, would appear to be not without some historical justification, which it is hoped to discuss in a later paper. There is reason to hope that a similar definition might also be adopted by the United States and thus end the present slight divergence between the yards of the two countries.

On this basis then we should have :—

$$1 \text{ Yard} = 1{,}552{,}734\cdot 81 \times 0\cdot 9144 = 1{,}419{,}820\cdot 71\ \lambda_{\mathrm{R}} \text{ in vacuum.}$$

Referring to Table XVII, it will be seen that the directly determined value of the Imperial Standard Yard is 1,419,818·31 λ_{R} in vacuum, which differs from the suggested value by 1·7 parts in 10^6.

Having once established the definitions of the units on the lines of some such scheme as suggested above, it is obvious that the material standards of length most suited for use in conjunction with these definitions are end-standards. The perfection which can now be attained in the parallelism and flatness of the hardened measuring faces of a steel end-standard not only affords a precision in the ascertainment of length which is definitely superior to that provided by line-standards, but at the same time permits the material standard to be directly verified in terms of the definition of the unit and to be employed in the practical measurement of length. Thus in a system of length measurement based on the methods and apparatus used in the present work, the X-gauge is the first material reference standard which is measured in terms of the definition and is then compared with other end-standards which are to be used in ordinary routine measurements of length. In addition, line-standards can be verified through the agency of the corresponding composite gauge, the length of which is also determined by comparison with the X-gauge. The results given in this paper show that the accuracy attainable in primary measurements of end-standards of the type described is at least five times greater than that now possible with line-standards.

In brief, the suggestions now advanced are as follows :—

(1) That the wave-length of some specified monochromatic radiation in vacuum, produced under suitably prescribed conditions, should be taken as the future basis of definition of the units of length.

(2) That the definition of the metre should be so chosen as to preserve the existing definition of the Angstrom unit on the basis $1\ \text{A} = 10^{-10}\ \text{M}$ and that the Angstrom should from thenceforth be re-defined as 10^{-10} M.

(3) That the yard should be defined in similar terms, on a basis which would make the yard equal to 0·9144 M.

(4) Assuming the adoption of the cadmium red radiation for the selected wave-length and the accuracy of the present investigation as regards refractive index of air, these suggestions lead to :—

$$1\ \text{M} = 1{,}552{,}734\cdot 81\ \lambda_{\text{R}}\ (\text{vac.})$$

$$1\ \text{Y} = 1{,}419{,}820\cdot 71\ \lambda_{\text{R}}\ (\text{vac.})$$

$$\lambda_{\text{R}}\ (\text{vac.}) = 6440\cdot 2498 \times 10^{-10}\ \text{M}.$$

Further investigations should, however, be made regarding the conditions for the uniform reproduction of a suitable source of monochromatic radiation, and to establish precise values of the refractive index of air under various conditions. Preparations are now being made for the study of both these questions at the National Physical Laboratory.

9. *Summary.*

Determinations of the lengths of the Prototype Metre and the Imperial Standard Yard in terms of the wave-length of the cadmium red radiation, both in air and in vacuum, have been made by methods and apparatus which have been previously described.

The results of these determinations have provided :—

(1) A new value for the length of the Metre, in terms of wave-lengths of the cadmium red radiation in air, which is in good agreement with the values obtained by other observers.

(2) The first directly determined value of the length of the Yard in terms of wave-lengths of the same radiation in air.

(3) The first directly determined values of the lengths of both units in terms of wave-lengths of the same radiation in vacuum.

(4) An incidental new value of the ratio of the yard to the metre which is in good agreement with the value directly determined by line-standard comparisons.

(5) An incidental new value of the refractive index of air for the red radiation of cadmium.

Evidence was further obtained that such hyperfine structure or asymmetry as may be present in the cadmium red radiation has no serious effect on the results of practical measurements of length of the type described.

As a consequence of this investigation it is possible to formulate certain tentative proposals for the future definition of the units of length on the basis of the wave-length of cadmium red radiation in vacuum, and it is shown that the precision of ascertainment of length on such a basis is at least five times greater than that now possible with line-standards.

10. *Acknowledgments.*

In conclusion, the authors desire to make the following acknowledgments: To Sir Joseph Petavel for his continued interest in this investigation and for permission to publish the results obtained; to Mr. R. F. Zobel, who very competently shared with the authors the immense amount of observational work entailed in the determinations of the X-gauges, in the comparisons of the X-gauges with the composite gauges and in the measurement of temperature, and who also assisted with the final computations, and to Miss W. M. Battersby for her valuable assistance in recording the observations and in carrying out the attendant calculations; to Mr. W. H. Johnson, who, with his assistants, Mr. L. O. C. Johnson and Mr. V. W. Stanley, undertook the important observational work involved in the comparisons of the composite gauges with the fundamental units of length on the line-standard comparator; to Mr. F. A. Gould, Mr. J. S. Clark, and Dr. J. C. Evans for their advice and help in connection with the absolute measurement of barometric pressure; to Mr. F. D. Jones and Mr. A. T. Powis who took all the observations on the barometer gauge, and again to Mr. V. W. Stanley, who assisted them in the comparison of this instrument with the Laboratory's Primary Standard Barometer; to Mr. J. A. Hall, of the Physics Department, and to Mr. S. Watts, of the Electrical Standards Department, for continuing the periodical re-calibrations of the platinum resistance thermometers and the standard resistance coils. Also to Dr. G. Barr, of the Metallurgy Department, for suggesting the method employed for removing water vapour and carbon dioxide from the samples of air used in the determinations; to the Superintendent of Research, Royal Arsenal, Woolwich, for undertaking the renewal of the chromium-plated surface of one end of étalon L_2 which had become necessary through progressive deterioration of the original plated surface; and finally to Mr. H. P. Bloxam for his assistance in connection with the construction and installation of the auxiliary apparatus mentioned in this paper.

APPENDIX I.

New Auxiliary Apparatus.

(*a*) *Air-conditioning Apparatus.*—Use was made of the recently introduced material known as silica gel for drying the air. This substance is believed to adsorb water vapour in a manner somewhat analogous to that of charcoal in adsorbing gases, and requires to be dehydrated before use. Dehydration is performed by passing a current of fairly dry air through the gel for 3 or 4 hours, the gel being heated to a temperature of about 300° C. Provided the gel is dehydrated at suitable intervals, it retains indefinitely its capacity for adsorbing moisture and therefore possesses a marked advantage over the usual drying agents.

Fig. 2 is a diagram showing the arrangement of the air-conditioning apparatus and the system of connections between the étalons, vacuum pump and barometer gauge.

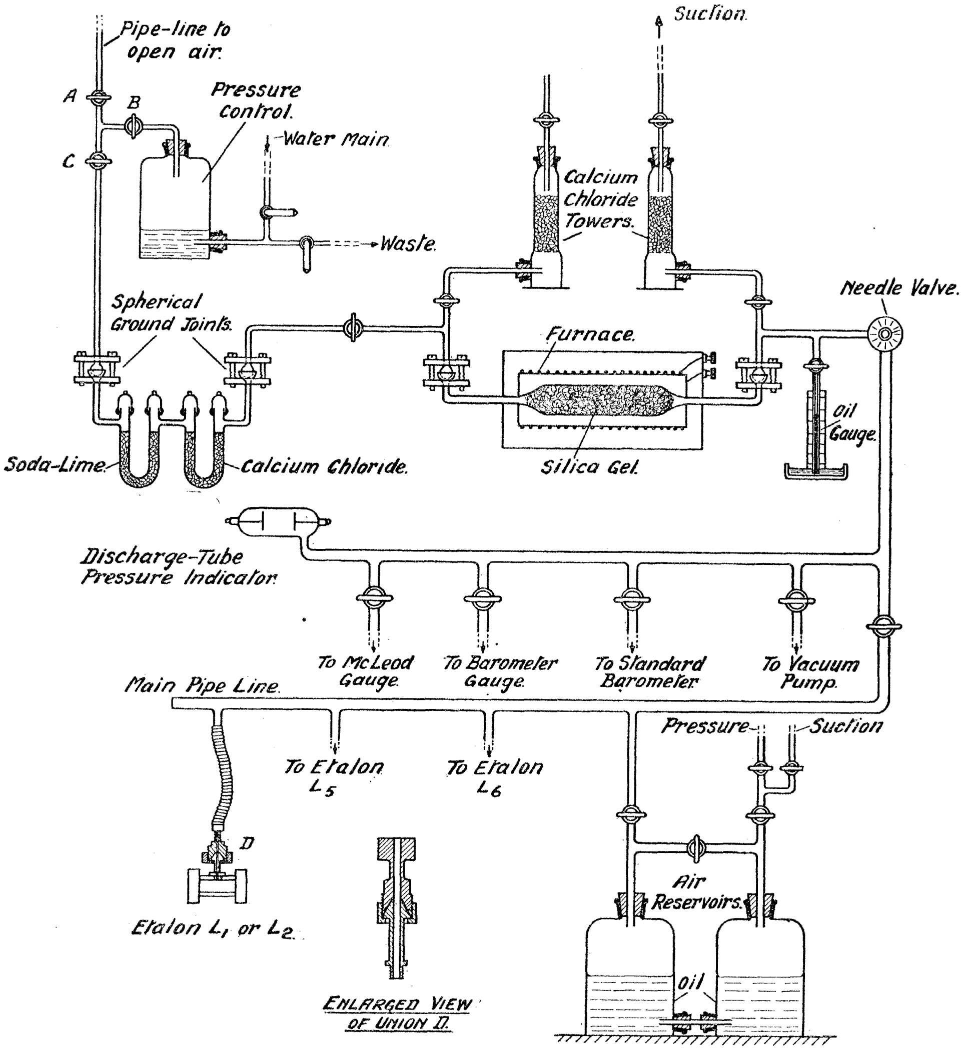

FIG. 2.—Diagram of the air-conditioning apparatus.

When the étalons and main pipe-line are evacuated air can be drawn into the apparatus by opening the needle valve. Air was not allowed to enter the apparatus from the room but was drawn from outside the building through a pipe-line which is seen connected to the apparatus at the top left corner of the diagram. It passed first through a U-tube containing Sofnolite, which is a brand of soda-lime containing an indicator that changes colour from green to brown upon absorption of carbon dioxide. Partial drying of the air was then performed by the U-tube containing calcium chloride, and the drying was completed in the silica gel tube. Spherical ground joints were fitted to the U-tubes to permit of their easy removal for purposes of refilling; similar joints were also provided for the silica gel tube.

In order to ensure an approximately constant flow of air through the apparatus during the filling of the étalon system, the opening of the needle valve was adjusted at suitable intervals during the process of filling. This adjustment was easily controlled by maintaining the column of oil in the gauge, seen at the left of the needle valve, at a height of about 5 to 6 cm., the flow of air then being at the rate of approximately 0·4 litre per minute.

The arrangements for dehydrating the silica gel are also shown in fig. 2. The tube containing the silica gel was mounted in an electrically heated furnace the temperature of which was thermostatically controlled at a value of about 300° C. A current of air was drawn from the room through the left calcium chloride tower and the silica gel tube by means of a water-jet pump connected to the top of the right calcium chloride tower. The latter tower was inserted to exclude vapour due to the water-jet pump from the silica gel tube.

Since it was desired to make the optical observations in air at pressures of about 760 mm. of mercury, special arrangements were made to adjust the pressure in the closed system when the atmospheric pressure was lower than 760 mm. An aspirator bottle was connected to the incoming pipe-line as shown in the top left corner of fig. 2. Commencing with the bottle full of water, stopcocks A and B being open and C closed, fresh air was drawn into the bottle by allowing water to escape. Stopcock A was then closed and C opened, and air was displaced into the air-conditioning apparatus and étalons by filling the bottle from the water main until the barometer gauge connected to the étalons indicated the appropriate reading, when the needle valve was closed. On occasions when the atmospheric pressure exceeded 760 mm. the adjustment to the standard pressure was simply made by closing the needle valve at the proper moment during the process of filling.

The air reservoirs shown at the bottom right-hand corner of fig. 2 were intended for fine adjustment of pressure, but this was found to be unnecessary for the present work. These reservoirs were situated in the same enclosure as the étalons and were therefore under precise temperature control. The increase in the volume of air at controlled temperature, obtained by the inclusion of the reservoirs with the étalons, was of great utility in reducing the effect of variations of room temperature (generally about ±0·1° C. during a single determination) upon the comparatively small volume of air in the pipe-line connecting the étalon system with the externally mounted barometer gauge.

A test of the efficiency of the drying arrangements was made by displacing a total volume of about 1100 litres of atmospheric air through the apparatus, and then finally through a weighed P_2O_5 tube. Before commencing the test the silica gel was dehydrated in the usual manner. The flow of air was maintained at about 0·6 litre a minute and after suitable intervals the P_2O_5 tube was weighed. The amount of water collected by the weighed tube from the first 100 litres was less than 1 mg., and the total amount collected for 1100 litres was 30 mg. The total amount of water vapour actually present in 1100 litres of unconditioned air under the existing conditions was about 9 gm. It was calculated that the average refractive index of the 1100 litres of air after passing through the silica gel differed from the refractive index of absolutely dry air by about 2 parts in 10^9. For the determinations of the yard and the metre in air the silica gel was dehydrated after the passage of every 100 litres of air.

(*b*) *Barometer Gauge.*—The barometer gauge used for the measurement of the pressure in the étalon system was designed and constructed at the Laboratory. A full description of the barometer will be published at a later date; the following is a brief account of its main features. The barometer is a Kew Pattern instrument fitted with a tube of bore 0·6-inch and a cistern of internal diameter 6 inches. Both inch and metric scales are provided, but only the metric scale was used for the purposes of this research. The vernier fitted to this scale is sub-divided so as to read directly to 0·02 mm., but readings to 0·01 mm. can readily be estimated. The temperature of the mercury column is determined by means of two thermometers placed one vertically above the other with their bulbs at approximately the same distance from the centre of the barometric column; the bulb of each thermometer is immersed in mercury contained in a cylindrical glass cup having the same radial dimensions as the barometer tube.

The barometer was standardized by reference to the Laboratory's Primary Standard Barometer,* comparisons of the two barometers being made at intervals throughout the period of time occupied by the optical measurements. During the actual optical observations, the barometer was maintained in direct connection with the étalon system, and the values of the pressure were determined at the beginning, in the middle and at the end of each cycle of observations. The mean of these values was taken as the pressure during the cycle.

The pressure measurements were shared by two skilled observers who have had special experience in barometric work. Each observer participated in the standardization of the barometer and thus determined the value of the index correction to be applied to the readings of the barometer taken by him during the course of the optical measurements. Details of the further corrections necessary in order to obtain the

---

* SEARS and CLARK, 'Proc. Roy. Soc.,' A, vol. 139, p. 130 (1933).

pressure in the étalon system in millimetres of mercury at 0° C. under standard conditions of gravity are given in Appendix II.

The value of the final mean pressure, both in the observations on the metre and in the observations on the yard, can be regarded as accurate to within ± 0·02 mm.

APPENDIX II.

Specimen Sets of Observations from the Optical Determinations.

(*a*) *Determination of an X-gauge.*—Table XIX displays the complete set of observations and attendant calculations applying to the determination in air, X_Y XXVII. The sequence of operations followed during the measurement of an X-gauge has been explained in the previous paper (*loc. cit.*, p. 119), from which it will be understood that all the required operations are accomplished in a certain order, which is subsequently repeated in inverse sense. In the table the observations and calculations relating to the first and second halves of the complete cycle are distinguished respectively by the sub-headings (*a*) and (*b*).

The observational work was commenced by taking readings on the platinum resistance thermometers and the barometer gauge; these readings are given in Table XIX, pp. 200–201, and are explained later in greater detail. The observer detailed for the optical measurements took no part in the accessory observations of temperature and pressure.

The first operation in the optical work consisted of the measurement of the diameters of five bright interference rings in the red radiation of cadmium, for the basic étalon L_1, in terms of divisions of the eyepiece micrometer fitted to the main telescope. The readings of the ten fringe settings necessary for this measurement are set down in column (1), under the heading "Fringe readings," in section (*a*) of the observations. They are obtained in one traverse of the crosshairs over the five chosen rings, commencing and ending on the outermost ring of the five. The horizontal gap at the middle of the ten readings indicates the centre of the concentric ring system. The micrometer settings were then repeated in inverse order, and the corresponding readings are set down in column (2), in which only the last two digits of the micrometer readings are given. The time at the mid-point of this complete series of readings is given at the corresponding place in the time column, and the mean values of the readings in columns (1) and (2) are shown in the mean column.

The observations in the red radiation of cadmium were followed immediately by similar observations in the auxiliary radiation, namely the green line of krypton. Only three rings in this radiation were measured, and only one traverse of the micrometer was made. In this case the time refers to the passage across the centre of the ring system.

It was usual to omit the diffuse innermost ring from the measurement of the diameters of systems of interference rings, attention being confined to the next three or five sharper rings outside it.

TABLE XIX.—Specimen Set of Observations and

Date :—May 30, 1933. Determination :—X_Y XXVII.

Operation.	Radiation.	Observations.							
		(*a*).				(*b*).			
		Time. a.m.	Fringe readings.			Time. a.m.	Fringe readings.		
			(1.)	(2.)	Mean.		(1.)	(2.)	Mean.
Determination of basic étalon L_1	Cadmium red		2895	94	2894		2896	97	2896
			2792	91	2792		2789	90	2790
			2673	68	2670		2667	71	2669
			2528	33	2530		2528	31	2530
			2357	54	2356		2358	58	2358
			1220	19	1220		1223	20	1222
			1053	54	1054		1053	55	1054
			917	19	918		920	19	920
			799	01	800		796	01	798
		10.14	696	94	695	11.03	696	95	696
	Krypton green		2608				975		
			2476				1099		
			2319				1249		
		10.19				10.59			
			1252				2316		
			1106				2480		
			978				2611		

Calculations for the Determination of an X-gauge.

Condition :—Air. Observer :—H. B.

Calculations.							
(*a*).				(*b*).			
Ring diameters (d).	$d^2 \times 10^{-3}$	$(p-1)d_p^2 \times 10^{-3}$	Calculation of excess fraction (ε).	Ring diameters (d).	$d^2 \times 10^{-3}$	$(p-1)d_p^2 \times 10^{-3}$	Calculation of excess fraction (ε).
2199 1992 1752 1476 1136	4835 3968 3070 2178 1291 15342 $=\Sigma$	19340 11904 6140 2178 39562 $=S$	$6\Sigma = 92052$ $2S = 79124$ 12928 $S = 39562$ $2\Sigma = 30684$ 8878 $\frac{12928}{8878} = 1{\cdot}456$	2200 1992 1749 1476 1136	4840 3968 3059 2178 1291 15336 $=\Sigma$	19360 11904 6118 2178 39560 $=S$	$6\Sigma = 92016$ $2S = 79120$ 12896 $S = 39560$ $2\Sigma = 30672$ 8888 $\frac{12896}{8888} = 1{\cdot}451$
Mean value of $\varepsilon = 0{\cdot}453_5$.							
1630 1370 1067	2657 1877 1139 5673 $=\Sigma$	5314 1877 7191 $=S$	$5\Sigma = 28365$ $3S = 21573$ 6792 $3S = 21573$ $3\Sigma = 17019$ 4554 $\frac{6792}{4554} = 1{\cdot}491$	1636 1381 1067	2677 1907 1139 5723 $=\Sigma$	5354 1907 7261 $=S$	$5\Sigma = 28615$ $3S = 21783$ 6832 $3S = 21783$ $3\Sigma = 17169$ 4614 $\frac{6832}{4614} = 1{\cdot}481$
Mean value of $\varepsilon = 0{\cdot}486$.							
$\varepsilon = 0{\cdot}453_5\ \lambda_R$. $2L_1 = 260{,}543{\cdot}453_5$. $4L_1 = 521{,}086{\cdot}907$.				$0{\cdot}486\ \lambda_{KG}$. $301{,}151{\cdot}527$.			

TABLE XIX—

Operation.	Radiation.	Observations.							
		(*a.*)				(*b.*)			
		Time. a.m.	Compensator readings.			Time. a.m.	Compensator readings.		
			(1.)	(2.)	Mean.		(1.)	(2.)	Mean.
Comparison of L_1 and L_5.	White light	10.21	1122·4 565·9	2·8 7·6	1122·6 566·8	10.56	567·0 1122·9	6·8 2·2	566·9 1122·6
					555·8 $=2\alpha$				555·7 $=2\alpha$
Comparison of L_5 and L_6	White light	Channel 1.							
		10.23	722·7 967·0	2·5 7·0	722·6 967·0	10.55	967·5 722·2	6·6 1·7	967·0 722·0
					244·4 $=2\beta$				245·0 $=2\beta$
		Channel 2.							
		10.24	965·7 724·2	6·0 4·1	965·8 724·2	10.54	725·0 964·1	4·8 3·9	724·9 964·0
					241·6 $=2\beta$				239·1 $=2\beta$
		Channel 3.							
		10.25	723·2 965·5	3·1 6·1	723·2 965·8	10.53	966·1 723·7	6·1 4·3	966·1 724·0
					242·6 $=2\beta$				242·1 $=2\beta$
		Channel 4.							
		10.26	965·2 724·6	5·2 5·5	965·2 725·0	10.52	724·1 965·3	4·9 4·6	724·5 965·0
					240·2 $=2\beta$				240·5 $=2\beta$

(*continued*)

Calculations.	
(*a*)	(*b*)
$2\alpha = 555\cdot8 \times 10^{-5}$ radian. $\frac{\alpha^2}{2} = \frac{30\cdot891}{8} \times 10^{-6} = 3\cdot861 \times 10^{-6}$.	$2\alpha = 555\cdot7 \times 10^{-5}$ radian. $\frac{\alpha^2}{2} = \frac{30\cdot880}{8} \times 10^{-6} = 3\cdot860 \times 10^{-6}$.
Mean value of $\alpha^2/2 = 3\cdot860_5 \times 10^{-6}$.	
$4L_1 = 521{,}086\cdot907\ \lambda_R$. $+ 4L_1 \times \alpha^2/2 = +2\cdot012$. $L_5 = 521{,}088\cdot919$ and $3L_5 = 1{,}563{,}266\cdot757\ \lambda_R$.	
$2\beta = 244\cdot4 \times 10^{-5}$ radian. $\frac{\beta^2}{2} = \frac{5\cdot974}{8} \times 10^{-6} = 0\cdot747 \times 10^{-6}$.	$2\beta = 245\cdot0 \times 10^{-5}$ radian. $\frac{\beta^2}{2} = \frac{6\cdot003}{8} \times 10^{-6} = 0\cdot750 \times 10^{-6}$.
Mean value of $\beta^2/2$ for channel $1 = 0\cdot748_5 \times 10^{-6}$.	
$2\beta = 241\cdot6 \times 10^{-5}$ radian. $\frac{\beta^2}{2} = \frac{5\cdot837}{8} \times 10^{-6} = 0\cdot730 \times 10^{-6}$.	$2\beta = 239\cdot1 \times 10^{-5}$ radian. $\frac{\beta^2}{2} = \frac{5\cdot717}{8} \times 10^{-6} = 0\cdot715 \times 10^{-6}$.
Mean value of $\beta^2/2$ for channel $2 = 0\cdot722_5 \times 10^{-6}$.	
$2\beta = 242\cdot6 \times 10^{-5}$ radian. $\frac{\beta^2}{2} = \frac{5\cdot885}{8} \times 10^{-6} = 0\cdot736 \times 10^{-6}$.	$2\beta = 242\cdot1 \times 10^{-5}$ radian. $\frac{\beta^2}{2} = \frac{5\cdot861}{8} \times 10^{-6} = 0\cdot733 \times 10^{-6}$.
Mean value of $\beta^2/2$ for channel $3 = 0\cdot734_5 \times 10^{-6}$.	
$2\beta = 240\cdot2 \times 10^{-5}$ radian. $\frac{\beta^2}{2} = \frac{5\cdot770}{8} \times 10^{-6} = 0\cdot721 \times 10^{-6}$.	$2\beta = 240\cdot5 \times 10^{-5}$ radian. $\frac{\beta^2}{2} = \frac{5\cdot784}{8} \times 10^{-6} = 0\cdot723 \times 10^{-6}$.
Mean value of $\beta^2/2$ for channel $4 = 0\cdot722 \times 10^{-6}$.	

Channel.	$3L_5 \times \beta^2/2$.
1	$1\cdot170\ \lambda_R$
2	$1\cdot129$
3	$1\cdot148$
4	$1\cdot129$
Mean	$1\cdot144\ \lambda_R$

$3L_5 = 1{,}563{,}266\cdot757\ \lambda_R$.
$- 3L_5 \times \beta^2/2 = -1\cdot144$
$L_6 = 1{,}563{,}265\cdot613$.

TABLE XIX—

Operation.	Radiation.	Observations.					
		(*a*)			(*b*)		
		Time a.m.	Fringe readings.		Time a.m.	Fringe readings.	
Determination of ($L_6 - X_Y$)	Krypton green	West.					
			2809	1160		1160	2805
			2729	1238		1239	2725
			2636	1324		1327	2637
			2530	1430		1435	2529
		10.27	2395	1561	10.50	1562	2399
		East.					
			2998	1284		1285	2998
			2912	1365		1368	2915
			2824	1451		1457	2823
			2719	1555		1559	2720
		10.29	2599	1682	10.49	1684	2598
	Cadmium green.	West.					
			1205	2771		2771	1200
			1282	2698		2698	1274
			1366	2605		2609	1357
			1456	2505		2513	1457
		10.30	1579	2387	10.46	2392	1571

(*continued*)

Calculations.							
(*a*)				(*b*)			
Ring diameters (*d*).	$d^2 \times 10^{-3}$	$(p-1)d_p^2 \times 10^{-3}$.	Calculation of excess fraction (ε).	Ring diameters (*d*).	$d^2 \times 10^{-3}$	$(p-1)d_p^2 \times 10^{-3}$.	Calculation of excess fraction (ε).
1649	2720	10880	$6\Sigma = 51420$	1645	2706	10824	$6\Sigma = 51156$
1491	2223	6669	$2S = 44402$	1486	2207	6621	$2S = 44146$
1312	1721	3442		1310	1716	3432	
1100	1210	1210	7018	1094	1196	1196	7010
834	696			837	701		
			$S = 22201$				$S = 22073$
	8570	22201	$2\Sigma = 17140$		8526	22073	$2\Sigma = 17052$
	$= \Sigma$	$= S$	5061		$= \Sigma$	$= S$	5021
			$\frac{7018}{5061} = 1\cdot387$				$\frac{7010}{5021} = 1\cdot396$
Mean value of $\varepsilon = 0\cdot391$.							
1714	2938	11752	$6\Sigma = 56478$	1713	2934	11736	$6\Sigma = 56262$
1547	2394	7182	$2S = 48118$	1547	2394	7182	$2S = 47996$
1373	1885	3770		1366	1866	3732	
1164	1355	1355	8360	1161	1348	1348	8266
917	841			914	835		
			$S = 24059$				$S = 23998$
	9413	24059	$2\Sigma = 18826$		9377	23998	$2\Sigma = 18754$
	$= \Sigma$	$= S$	5233		$= \Sigma$	$= S$	5244
			$\frac{8360}{5233} = 1\cdot598$				$\frac{8266}{5244} = 1\cdot576$
Mean value of $\varepsilon = 0\cdot587$.							
1566	2453	9812	$6\Sigma = 46482$	1571	2468	9872	$6\Sigma = 47124$
1416	2005	6015	$2S = 39996$	1424	2028	6084	$2S = 40416$
1239	1535	3070		1252	1568	3136	
1049	1101	1101	6486	1056	1116	1116	6708
808	653			821	674		
			$S = 19998$				$S = 20208$
	7747	19998	$2\Sigma = 15494$		7854	20208	$2\Sigma = 15708$
	$= \Sigma$	$= S$	4504		$= \Sigma$	$= S$	4500
			$\frac{6486}{4504} = 1\cdot440$				$\frac{6708}{4500} = 1\cdot491$
Mean value of $\varepsilon = 0\cdot465$.							

Table XIX—

Operation.	Radiation.	Observations.					
		(a)			(b)		
		Time a.m.	Fringe readings.		Time a.m.	Fringe readings.	
Determination of ($L_6 - X_Y$) (continued)	Cadmium green (continued)	East.					
			1322	2955		2952	1320
			1402	2873		2879	1395
			1493	2785		2788	1481
			1590	2683		2678	1583
		10.32	1708	2563	10.45	2555	1694
	Cadmium red	West.					
			2849	1125		1124	2846
			2757	1210		1216	2756
			2656	1303		1312	2659
			2540	1422		1427	2539
		10.34	2380	1569	10.43	1570	2385
		East.					
			3069	1205		1202	3066
			2981	1290		1290	2979
			2883	1383		1385	2883
			2775	1491		1498	2769
		10.36	2641	1619	10.42	1622	2641

(*continued*)

Calculations.							
(*a*)				(*b*)			
Ring diameters (*d*).	$d^2 \times 10^{-3}$.	$(p-1)\,d_p^2 \times 10^{-3}$.	Calculation of excess fraction (ε).	Ring diameters (*d*).	$d^2 \times 10^{-3}$.	$(p-1)\,d_p^2 \times 10^{-3}$.	Calculation of excess fraction (ε).
1633	2667	10668	6Σ = 50550	1632	2664	10656	6Σ = 51084
1471	2164	6492	2S = 43384	1484	2202	6606	2S = 43756
1292	1669	3338		1307	1709	3418	
1093	1194	1194	7166	1095	1198	1198	7328
855	731			861	741		
			S = 21692				S = 21878
	8425	21692	2Σ = 16850		8514	21878	2Σ = 17028
	= Σ	= S	4842		= Σ	= S	4850
			$\frac{7166}{4842}=1{\cdot}480$				$\frac{7328}{4850}=1{\cdot}511$
Mean value of ε = 0·495.							
1724	2972	11888	6Σ = 54630	1722	2965	11860	6Σ = 54318
1547	2394	7182	2S = 47964	1540	2372	7116	2S = 47686
1353	1831	3662		1347	1815	3630	
1118	1250	1250	6666	1112	1237	1237	6632
811	658			815	664		
			S = 23982				S = 23843
	9105	23982	2Σ = 18210		9053	23843	2Σ = 18106
	= Σ	= S	5772		= Σ	= S	5737
			$\frac{6666}{5772}=1{\cdot}155$				$\frac{6632}{5737}=1{\cdot}156$
Mean value of ε = 0·155.							
1864	3475	13900	6Σ = 67656	1864	3475	13900	6Σ = 67344
1691	2859	8577	2S = 57250	1689	2852	8556	2S = 57114
1500	2250	4500		1498	2243	4486	
1284	1648	1648	10406	1271	1615	1615	10230
1022	1044			1019	1039		
			S = 28625				S = 28557
	11276	28625	2Σ = 22552		11224	28557	2Σ = 22448
	= Σ	= S	6073		= Σ	= S	6109
			$\frac{10406}{6073}=1{\cdot}713$				$\frac{10230}{6109}=1{\cdot}675$
Mean value of ε = 0·694.							

ε (West)	0·155 λ_R.	0·391 λ_{KG}.	0·465 λ_G.
ε (East)	0·694	0·587	0·495
ε (W + E)	0·849	0·978	0·960
2 $(L_6 - X_Y)$	286,053·849	330,637·950	362,134·047

$(L_6 - X_Y) = 143{,}026{\cdot}924$

$L_6 = 1{,}563{,}265{\cdot}613$

$X_Y = 1{,}420{,}238{\cdot}689$

TABLE XIX—

Temperature observations. Observer : R. F. Z.

Room temp. (° C.).	Time a.m.	Bath temp. (° C.).	Potentiometer dial readings.				
			S.	T_2.	Θ_E.	Θ_W.	T_1.
	10.04	20·67	50—51	45—18	81—82	80—03·5	49—82
		20·67	50—49·5	45—17	81—81	80—03	49—81·5
West : 20·9		20·67					
	10.37	20·67	50—49·5	45—19	81—79	80—01	49—83·5
		20·67	50—49	45—18	81—78·5	80—00	49—83
East : 20·9		20·67					
	11.06	20·67	50—48	45—18	81—77	79—99·5	49—83
		20·67	50—47	45—17·5	81—78	79—99·5	49—82·5
		20·66					
Mean		20·67	50—49·0	45—17·9	81—79·2	80—01·1	49—82·6

Temperature calculations.

Standard resistance S.

Fixed coil . . .	250027·5	Bath temperature (corrd.)	= 20·63° C.
Dials (corrd.) .	5048·6	Standard coil L 22428	= 99·9973 ohms.
		„ „ L 22432	= 99·9962
	255076·1		
		Mean	= 99·9967$_5$
		S	= 49·9984

Thermometer T_2.

Fixed coil . . .	250027·5	R 49·8944 ohms.	T_2 22·391° Pt.
Dials (corrd.) .	4517·7	R_0 45·8854	—0·012
	254545·2	4·0090	22·379
R/S	0·997,919	F.I./100 0·179,049	=22·121° C.

Thermometer Θ_E.

Fixed coil . . .	250027·5	R 50·6118 ohms.	Θ_E 22·379° Pt.
Dials (corrd.) .	8178·2	R_0 46·5319	=22·121° C.
	258205·7	4·0799	
R/S	1·012,269	F.I./100 0·182,309	

Thermometer Θ_W.

Fixed coil . . .	250027·5	R 50·5770 ohms.	Θ_W 22·380° Pt.
Dials (corrd.) .	8000·4	R_0 46·5006	=22·122° C.
	258027·9	4·0764	
R/S	1·011,572	F.I./100 0·182,146	

Thermometer T_1.

Fixed coil . . .	250027·5	R 49·9854 ohms.	T_1 22·395° Pt.
Dials (corrd.) .	4981·9	R_0 45·9732	—0·012
	255009·4	4·0122	22·383
R/S	0·999,739	F.I./100 0·179,158	=22·125° C.

T_1.	Θ.	T_2.
22·125° C.	22·121$_5$° C.	22·121° C.

(*continued*)

Pressure measurements. Observer :—F. D. J.

Time.	Corrected temperatures.		Barometer reading.
	Upper thermometer.	Lower thermometer.	
a.m.	° C.	° C.	mm.
10.04	$20\cdot6_5$	$20\cdot6_3$	$762\cdot26_5$
10.37	$20\cdot6_9$	$20\cdot6_8$	$762\cdot29_0$
11.06	$20\cdot7_4$	$20\cdot7_1$	$762\cdot30_5$
Mean	$20\cdot6_8$		$762\cdot28_7$

Mean barometer reading . . . . = $762\cdot28_7$
Index correction . . . . = $+\,0\cdot13_8$
Temperature correction . . . . = $-\,2\cdot78_9$
Altitude correction . . . . = $-\,0\cdot02_1$
Gravity correction . . . . = $+\,0\cdot41_0$
Pressure in $\begin{cases}\text{millimetres of mercury at 0° C.,}\\ g = 980\cdot665 \text{ cm. per sec. per sec.}\end{cases}$ . . . . = 760·03

Reduction of observed value of X_Y to the mean conditions of all observations of X_Y in air.

Determination No.	Observer.	Basic étalon.	T_1 (° C.).	Θ (° C.).	Pressure (h. mm.).	Observed value of X_Y (λ_R).
X_Y XXVII	H.B.	L_1	22·125	$22\cdot121_5$	760·03	1,420,238·689
Basic conditions			$21\cdot351_5$	$21\cdot351_5$	760·00	

Refractive index correction.

Determination of L_6.

$$N_m - N_a = 1\cdot5633\ [270\cdot336 - 0\cdot383{,}928\ h/(1 + 0\cdot003716\ T_1)]$$
$$= \qquad + 1\cdot5633 \times 0\cdot707 \qquad = + 1\cdot105\ \lambda_R.$$

Determination of $(L_6 - X_Y)$.

$$n_m - n'_a \quad = 0\cdot1430\ [270\cdot336 - 0\cdot383{,}928\ h/(1 + 0\cdot003716\ \Theta)]$$
$$= \qquad + 0\cdot1430 \times 0\cdot704 \qquad = + 0\cdot101\ \lambda_R.$$

Refractive index correction $= 1\cdot105 - 0\cdot101 = + 1\cdot004\ \lambda_R$.

Temperature correction.

Coefficient of expansion of $X_Y = (10\cdot531 + 0\cdot00474 \times 2t)\ 10^{-6}$
$= 10\cdot737 \times 10^{-6}$ at $t = \frac{1}{2}\,(21\cdot351_5 + \Theta)$ or $21\cdot736_5$° C.

Temperature correction $= 10\cdot737\ .\ X_Y\ .\ (21\cdot351_5 - \Theta)\ 10^{-6} = - 10\cdot737 \times 1\cdot4202 \times 0\cdot770$
$= - 11\cdot742\ \lambda_R$.

Total correction $= - 11\cdot742 + 1\cdot004 = - 10\cdot738\ \lambda_R$.

X_Y at $21\cdot351_5$° C. $= 1{,}420{,}227\cdot951\ \lambda_R$ in air at $21\cdot351_5$ ° C.

The measurement of the diameters of the circular fringes produced by L_1 was followed by the comparison of L_1 with L_5, for which the two étalons were illuminated by white light. The quantity to be measured in this comparison was the angle through which the axis of L_5 had to be inclined in a vertical plane, with respect to the optical axis of the main telescope, in order to bring the central white fringe of the system of fringes of superposition (or BREWSTER's fringes) into coincidence with the horizontal, fixed cross-wire of the telescope. The inclination was first performed in one sense, the central fringe being identified in white light, and the final setting made in red light obtained by interposing a Wratten filter, No. 26, in the beam of white light. The compensating micrometer was then read and a repeated setting and reading made. In Table XIX these two readings are shown respectively in columns (1) and (2), under the heading "Compensator readings," only the last two digits of the repeated compensator reading being given in column (2). A similar pair of readings, entered likewise in the table, was subsequently obtained by inclining L_5 in the opposite sense. The reading in the time column indicates the time when the inclination was reversed. Mean values of the two pairs of readings are given in the mean column, which shows also the value of 2α, or the difference between the mean readings in the two senses of inclination. The unit of measurement is 0·00001 radian.

In the same manner L_5 was compared with L_6, each one of the four optical channels of the latter being separately compared with the former. The compensator and time readings are also entered in the table in a similar manner and the values of 2β for each of the four channels are derived in the same way as 2α above.

Then followed the measurements of the West and East gaps between adjacent end-faces of the X-gauge and L_6, which are required for the determination of (L_6 — X_Y). The observations were similar to those made in the determination of L_1, but here the circular rings were observed in reflected light, so that the settings were made on five rings of the system of dark rings seen through the micrometer eye-piece which is fitted to an auxiliary telescope opposite each gap. Observations on the West and East gaps are appropriately distinguished in the table. The micrometer settings were obtained in one traverse only across the rings and the two readings at opposite ends of a diameter of a certain ring are entered horizontally in the table, while the time of passage across the centre of each ring system is suitably entered in the time column. The diameter measurements were made alternately at the West and East gaps in three monochromatic radiations, namely, krypton green, cadmium green, and cadmium red, in the order given.

At this stage the platinum thermometers and barometer gauge were read a second time. Then the optical observations described above were repeated in reverse order, the readings being entered in section (*b*) of the table in the same manner as in section (*a*). Final readings of the platinum resistance thermometers and barometer gauge were subsequently obtained, and these completed the cycle of observations necessary for a definitive determination of the X-gauge.

Excess fractions for the two radiations in L_1 were calculated from the measured ring diameters by means of a least squares method (ROLT and BARRELL, *loc. cit.*). The general expression for calculating the excess fraction ε from the measured linear diameters of p rings is :—

$$\varepsilon = \frac{\sigma\Sigma - sS}{pS - s\Sigma}$$

where :—

$$\Sigma = d_1^2 + d_2^2 + d_3^2 + \ldots\ldots\ldots\ldots + d_p^2,$$

$$S = d_2^2 + 2d_3^2 + 3d_4^2 + \ldots\ldots\ldots\ldots + (p-1)\,d_p^2,$$

$$s = 1 + 2 + 3 + \ldots\ldots\ldots\ldots + (p-1),$$

$$\sigma = 1^2 + 2^2 + 3^2 + \ldots\ldots\ldots\ldots + (p-1)^2,$$

d_1, d_2, d_3, ..., d_p are the linear diameters, in any arbitrary units, of the first p rings counting outwards from the centre of the system. If the innermost ring is omitted from the measurements, the same expression, when applied to the measured diameters of the next p rings, automatically gives the value of $(1 + \varepsilon)$. If three or five rings are measured the alternative expressions for ε become :—

$$\varepsilon = \frac{5\Sigma - 3S}{3S - 3\Sigma} \text{ for 3 rings.}$$

$$\varepsilon = \frac{6\Sigma - 2S}{S - 2\Sigma} \text{ for 5 rings.}$$

The first columns of the (*a*) and (*b*) sections of the calculations give the ring diameters which are derived from the mean readings shown in the corresponding sections of the observations. The second columns show the squares of the diameters, rounded off to four significant figures, the sum of which is the value of Σ in the expressions for ε given above, while the third columns show the terms $(p - 1)d_p^2$ which, added together, give the value of S. Details of the calculation of ε are shown in the fourth columns, together with the calculated values of $(1 + \varepsilon)$.

The mean values of ε for cadmium red (λ_R) and krypton green (λ_{KG}), derived from the two halves of the cycle, were then employed to check the order of interference for λ_R in L_1 by the usual method of coincidences of excess fractions. It should be mentioned that preliminary determinations of this order of interference had been made by the methods previously described. The preliminary measurements indicated an approximate order of 260,543 for λ_R in L_1. In the table it will be seen that this number is combined with the observed excess fraction for λ_R. The product of this combined order and the ratio λ_R/λ_{KG} gave the calculated value of the order shown for λ_{KG}, in which the excess fraction satisfactorily agrees with the observed value. The accepted value of the ratio λ_R/λ_{KG}, using the values of λ_R and λ_{KG} in air given in Table III, was 1·155,859,12. Having thus derived the value of $2L_1$ in terms of λ_R, the value of $4L_1$ was calculated.

The comparison of L_1 and L_5 by the method of optical multiplication supplied the value of α in the expression :—

$$L_5 \cos \alpha = 4L_1$$

where α is half the difference between the mean compensator readings. Since α was small, then :—

$$L_5 = 4L_1 \ (1 + \alpha^2/2).$$

The two values of $\alpha^2/2$ were calculated from the two measured values of 2α obtained in the two halves of the cycle. Then the product of the mean value of $\alpha^2/2$ and $4L_1$ gave the correction which, added to $4L_1$, produced the value of L_5, from which $3L_5$ was evaluated.

In like manner the four values of $3L_5 \times \beta^2/2$, corresponding to the four channels of L_6, were calculated. But in these operations the comparison of L_5 and L_6 gave the value of β in the relation :—

$$L_6 = 3L_5 \ (1 - \beta^2/2).$$

Therefore the mean value of the four corrections was subtracted from the value of $3L_5$ to produce the value of L_6.

The calculations of the excess fractions for the West and East gaps comprising $(L_6 - X_Y)$ followed the same lines as those already outlined for the basic étalon. The series of excess fractions for the two gaps were then added together and the method of coincidences applied to the combined values so obtained. A preliminary measurement of the order of interference for λ_R in $(L_6 - X_Y)$ at a particular temperature had been previously made by the usual methods, from which the expected value of the order at the existing temperature was calculated and found to be 286,054 approximately. The accepted value of the ratio λ_R/λ_{KG} in air has already been quoted, and the value of λ_R/λ_G in air, derived from Table III, is 1·265,964,60. It will be seen that the agreement between the observed and calculated excess fractions for λ_{KG} and λ_G was close enough to eliminate the possibility of the accepted order for λ_R being in error by one unit in its integral part. Subtracting the value of $(L_6 - X_Y)$ from the value of L_6, the length of X_Y in terms of λ_R was obtained.

Measurements of the resistances of the platinum thermometers were made by potentiometer comparison with a standard resistance, as explained in Appendix II of the previous paper. Each thermometer has a resistance of approximately 50 ohms at 20° C. The standard resistance consists of two 100-ohm coils connected in parallel and immersed in a stirred paraffin bath, the temperature of which was measured by a mercury-in-glass thermometer. A Tinsley thermo-electric potentiometer, modified by the inclusion of a 100-ohm fixed coil, was used for the comparison of the standard resistance with the resistances of the thermometers. The two dials were each composed of 100 studs, the difference of resistance for 1 stud on the right-hand dial being 0·0004-ohm, which is approximately equal to the change of resistance of the 50-ohm thermometers per 0·001° C. Where necessary the potentiometer readings were estimated to the nearest half stud of the right-hand dial.

Referring to the section of temperature observations on p. 200, each of the three series of readings commenced with an observation of the bath temperature, followed by observations, taken usually at the rate of one every 30 seconds, of the potentiometer dials for the standard resistance S and for each of the thermometers in turn. Both dial readings are suitably recorded in the table. Then the thermometer current of 0·004-amp. was reversed, in order to eliminate the effect of thermo-electric e.m.f.s., and a second reading of the bath temperature was obtained. The potentiometer comparisons were subsequently repeated in reversed order and the bath temperature read again. The mean values of the bath temperatures and the dial readings are shown at the foot of the columns of observations.

The room temperatures given in the table are the temperatures indicated by two mercury-in-glass thermometers, situated at each end of the roof of the enclosure surrounding the optical apparatus, which were read only at the beginning of the set of observations.

In the calculations of temperature the following equations were accepted for the four thermometers, Θ_E, Θ_W, T_1, and T_2, used during the present work :—

$$R\ (\Theta_E) = 46 \cdot 5319 + 0 \cdot 182{,}309\ T^\circ\ \text{Pt.}$$

$$R\ (\Theta_W) = 46 \cdot 5006 + 0 \cdot 182{,}146\ T^\circ\ \text{Pt.}$$

$$R\ (T_1) = 45 \cdot 9732 + 0 \cdot 179{,}158\ T^\circ\ \text{Pt.}$$

$$R\ (T_2) = 45 \cdot 8854 + 0 \cdot 179{,}049\ T^\circ\ \text{Pt.}$$

The equations for Θ_E and Θ_W were derived from the calibration of Θ performed in March, 1930, and described in Appendix II of the previous paper. Comparisons of Θ with T_1 and T_2 in the constant temperature enclosure had shown that this calibration was still correct to within 0·001° C. at ordinary temperatures. The equations for T_1 and T_2 were derived from a re-calibration made on March 1st, 1933.

T_1 and T_2 were calibrated again on September 1st, 1933, after the determinations of the metre and the yard had been completed. The new equations were :—

$$R\ (T_1) = 45 \cdot 9731 + 0 \cdot 179{,}159\ T^\circ\ \text{Pt.}$$

$$R\ (T_2) = 45 \cdot 8856 + 0 \cdot 179{,}049\ T^\circ\ \text{Pt.}$$

The changes over a period of nearly six months were very small, being negligible for T_1 and amounting to about 0·001° C. at 20° C. for T_2. All temperatures by T_1 and T_2 were worked out on the basis of the earlier calibration, but since the readings of T_2 do not enter into the optical calculations no alteration of the basis of temperature adjustment of the final results for the metre and the yard was necessary on this account.

The values of the two standard resistance coils at 20° C. in terms of the International Ohm were, on January 31st, 1933 :—

L 22428	99·9959 ohms.
L 22432	99·9948 ohms.

and on July 1st, 1933 :—

L 22428	99·9961 ohms.
L 22432	99·9947 ohms.

Here again the changes were negligible, and the calculations of temperature based on the earlier values were unaffected by the later values.

In the calculations of temperature it will be seen that the values of the fixed potentiometer coil and the dials are expressed in units of 0·0004-ohm, where the dial readings are corrected for small errors determined by previous calibration. The values of the two 100-ohm standard coils at the corrected temperature of 20·63° C. were obtained from resistance-temperature charts of the coils supplied by the Electrical Standards Department of the Laboratory from measurements of the resistances of the coils over a temperature range extending from 15° C. to 29° C. The platinum temperatures obtained from the readings of T_1 and T_2 had to be corrected by — 0·012° Pt., of which — 0·010° Pt. was due to the difference in the heating effect of a current of 0·004-amp. on the thermometer wire under the conditions of calibration in the bath of stirred water and under the conditions of use in the stirred air of the apparatus enclosure. The additional correction of — 0·002° Pt. arises from the fact that a comparison at 50° C. of the indications of T_1 and T_2 with those of standard thermometers of known δ showed that the value of δ for T_1 and T_2 was somewhat higher than that for Θ. A correction of — 0·002° Pt. is sufficient to adjust this difference at 20° C. Platinum temperatures were corrected to values in terms of degrees Centigrade on the International Temperature Scale by means of a conversion table, based on the usual formula $(t - t_{pt}) = \delta . t\ (t - 100)\ 10^{-4}$ for a value of $\delta = 1{\cdot}500$, which was supplied by the Physics Department of the Laboratory.

Readings of the barometer gauge and its associated thermometers are shown in the section of Table XIX devoted to pressure measurements. The index correction has already been referred to in Appendix I. The reduction to the standard temperature of 0° C. was made in the usual manner, account being taken of the fact that the barometer is a Kew Pattern instrument. The difference of level between the étalon system and the cistern of the gauge barometer was 9 inches and a correction of — 0·021 mm. was accordingly applied to the mean reading of the barometer. As the value of gravity at the Laboratory is 981·195 cm. per sec. per sec., the correction to reduce a barometer reading h at the Laboratory to the standard gravity value 980·665 cm. per sec. per sec. is $+ 0{\cdot}530\ h/980{\cdot}665$.

The last portion of Table XIX gives a summary of the determination No. XXVII of X_Y, and includes all the information necessary for the reduction of the observed value of X_Y to a value corresponding to the mean conditions of all observations of X_Y in air. The corrections for refractive index and temperature have already been discussed in the main part of this paper. The value of t substituted in the expression for the coefficient of expansion of X_Y is the mean value of the observed temperature Θ and the basic temperature of reduction for X_Y of $21 \cdot 351_5$° C.

Determinations in vacuum were carried out in a similar way to that described above for a determination in air, except that measurements of the residual pressure of air in the étalon system, by means of a discharge tube indicator or a McLeod Gauge, were substituted for the measurements of ordinary pressures by the barometer gauge. As already indicated the average correction for the residual air pressure amounted only to about $- 0 \cdot 01\ \lambda_R$.

(*b*) *Comparison of an X-gauge with a Composite Gauge.*—The operation of comparing two gauges in the End-gauge Comparator can be regarded as a simple extension of the determination of the difference in length between étalon L_6 and an X-gauge. In this comparator the gauges and their supports are mounted between two semi-silvered mirrors which are independently adjustable into parallelism with the end-faces of the gauge, the supporting arrangements ensuring that the axes of the gauges are parallel to one another and, therefore, that the four end-faces are also mutually parallel. If the lengths of the two gauges under comparison are represented by M' and X_M respectively and if the semi-silvered mirrors are separated by a distance C, which, by reason of the design of the comparator, may be regarded as constant apart from the effect of temperature changes, then :—

$$M' - X_M = (C - X_M) - (C - M').$$

The measurements of $(C - X_M)$ and $(C - M')$ were made in a manner similar to that already described for $(L_6 - X_Y)$. Thus, for instance, X_M was first introduced between the mirrors and measurements of the ring diameters in two or more monochromatic radiations were made at the gap at each end; then M' and X_M were interchanged so that M' was placed in the position formerly occupied by X_M, and similar measurements were made at the gap at each end of M'.

Compensation for the effect on the basic distance C of linear variations of temperature with time was obtained by observing the four gaps first in the order X_M-West, X_M-East, M'-East, and M'-West, and then in the reversed order. As it was not possible to make the comparisons under precisely controlled conditions of temperature it was also found necessary to take the measurements of the ring diameters in the series of monochromatic radiations at each gap in cyclical order, so that the mean values of the excess fractions calculated from the ring diameters measured in each half of the cycle all applied to the magnitude of the gap at a definite time, assuming of course that the variations of temperature were linear with time.

Table XX is a specimen set of observations and calculations applying to the comparison, No. III (*a*), of X_M with the composite gauge M'. The disposition of the two

TABLE XX.—Specimen Set of Observations and Calculations for the Comparison of an X-Gauge and a Composite Gauge.

Date :—June 13, 1933. Comparison of X_M and M′, No. III (*a*). Observer :—R. F. Z.

Observed values of the excess fractions.

	Radiation.	X_M—West.			X_M—East.			X_MW+X_ME.	M′—East.			M′—West.			M′E+M′W.
		(1).	(2).	Mean.	(1).	(2).	Mean.		(1).	(2).	Mean.	(1).	(2).	Mean.	
A	λ_R	0·021	0·003	0·012	0·901	0·923	0·912	0·924	0·474	0·421	0·447	0·758	0·768	0·763	0·210
	λ_{KY}	0·621	0·605	0·613	0·585	0·600	0·592	0·205	0·210	0·192	0·201	0·527	0·483	0·505	0·706
	λ_{KG}	0·530	0·542	0·536	0·518	0·524	0·521	0·057	0·949	0·925	0·937	0·373	0·457	0·415	0·352
	λ_{MG}	0·180	0·174	0·177	0·355	0·379	0·367	0·544	0·203	0·215	0·209	0·278	0·256	0·267	0·476
	λ_G	0·030	—	0·030	0·311	—	0·311	0·341	0·732	—	0·732	0·879	—	0·879	0·611
B	λ_R	0·164	0·155	0·159	0·614	0·640	0·627	0·786	0·571	0·654	0·612	0·490	0·456	0·473	0·085
	λ_{KY}	0·307	0·311	0·309	0·752	0·803	0·777	0·086	0·676	0·668	0·672	0·908	0·899	0·903	0·575
	λ_{KG}	0·422	0·420	0·421	0·382	0·430	0·406	0·827	0·008	0·071	0·039	0·110	0·221	0·165	0·204
	λ_{MG}	0·248	0·240	0·244	0·086	0·106	0·096	0·340	0·169	0·157	0·163	0·168	0·205	0·186	0·349
	λ_G	0·502	—	0·502	0·588	—	0·588	0·090	0·091	—	0·091	0·379	—	0·379	0·470

Temperature of comparison :—T_1 (mean) 22·212° C., T_2 (mean) 22·221° C., ∴ T = 22·216° C.

Calculation of $(M' - X_M)$.

	A.					B.				
	λ_R.	λ_{KY}.	λ_{KG}.	λ_{MG}.	λ_G.	λ_R.	λ_{KY}.	λ_{KG}.	λ_{MG}.	λ_G.
$\varepsilon\,(X_MW + X_ME)$	0·924	0·205	0·057	0·544	0·341	0·786	0·086	0·827	0·340	0·090
$\varepsilon\,(M'E + M'W)$	0·210	0·706	0·352	0·476	0·611	0·085	0·575	0·204	0·349	0·470
$\varepsilon\,(M' - X_M)$	0·714	0·499	0·705	0·068	0·730	0·701	0·511	0·623	0·991	0·620
$2\,(M' - X_M)$	18·714	20·523	21·631	22·065	23·691	18·701	20·509	21·616	22·049	23·675
$M' - X_M$	9·357	—	—	—	—	9·350	—	—	—	—

Mean value of $(M' - X_M)$ at 22·216° C. = 9·354 λ_R.

Reduction of value of $(M' - X_M)$ to temperature basis of 21·005° C.

Coefficient of expansion of M′ = $(10{\cdot}540 + 0{\cdot}00677 \times 2t)\ 10^{-6} = 10{\cdot}833 \times 10^{-6}$ at $t = (21{\cdot}005 + T)/2 = 21{\cdot}610°$ C.

Coefficient of expansion of X_M = $(10{\cdot}494 + 0{\cdot}00595 \times 2t)\ 10^{-6} = 10{\cdot}751 \times 10^{-6}$ at $t = 21{\cdot}610°$ C.

Correction to value of $(M' - X_M) = (10{\cdot}833 - 10{\cdot}751)\ (21{\cdot}005 - T)\ X_M \times 10^{-6} = -0{\cdot}082 \times 1{\cdot}211 \times 1{\cdot}553\ \lambda_R = -0{\cdot}154\ \lambda_R$

∴ $(M' - X_M)$ at 21·005° C. = 9·200 λ_R.

gauges for this comparison can be seen in fig. 1 (*c*). Since the relevant observations of ring diameters were similar to those already shown in Table XIX, they have been omitted from Table XX. The calculations of the excess fractions have also been omitted, and it is sufficient to mention that the values of the fractions were derived by the least squares method from measurements of the linear diameters of three rings in each radiation.

The values of the excess fractions in Table XX are arranged into two main groups, A and B, corresponding to the two halves of the main cycle of observations. The comparison was commenced by taking readings of the two platinum thermometers T_1 and T_2. Then followed the measurements of the ring diameters in a series of five radiations at the gap X_M—West, the five radiations in the order used being cadmium red (λ_R), krypton yellow (λ_{KY}), krypton green (λ_{KG}), mercury green (λ_{MG}), and cadmium green (λ_G). The values of the excess fractions for these radiations, calculated from the measured ring diameters, are set down in column (1) under the sub-heading X_M—West. The measurements were then repeated in reversed order, omitting the second measurements in λ_G in order to save time, and the values of the fractions derived from the repeated measurements are entered in column (2) under the same sub-heading, while the mean values of the two series of fractions are shown in the mean column.

Similar observations were afterwards made at the gap, X_M—East, at the other end of X_M, and the fractions obtained are entered similarly in the table. The fractions under the sub-heading ($X_M W + X_M E$) in the table were then obtained by adding together the two columns of mean fractions corresponding to the gaps X_M—West and X_M—East.

In the same manner measurements were made at the gaps M′—East and M′—West, and the fractions corresponding to the sum of the gaps were derived as before.

The platinum thermometers were then read a second time, after which the optical observations outlined above were repeated in reversed order and final readings of the thermometers were taken.

In calculating the difference in length between M′ and X_M, first the differences between the fractions in each radiation for the combined gaps ($X_M W + X_M E$) and ($M'E + M'W$) were evaluated. It was previously ascertained by ordinary mechanical measurements that M′ was longer than X_M so that the sense in which the differences were to be taken was already known. At the same time the mechanical measurements supplied an approximate value of the magnitude of ($M' - X_M$) which was required for calculating an approximate value of the corresponding order of interference in λ_R. Then the method of coincidences of excess fractions was applied to the series of difference fractions for both the A and B sections of the main cycle.

The values of $2(M' - X_M)$ were obtained by multiplying the adopted value in terms of λ_R in each section by the series of known ratios λ_R/λ_{KY}, λ_R/λ_{KG}, λ_R/λ_{MG}, and λ_R/λ_G, the accepted values of the wave-lengths of the five radiations being given in Table III. It will be seen that the adopted values in terms of λ_R were completely established by reason of the satisfactory agreement found between calculated and observed values of the difference fractions in the four auxiliary radiations.

The potentiometer observations and the calculation of temperatures have been omitted from the table and only the mean results of the three series of temperature readings are given.

The reduction of the observed value of (M′ — X_M), in terms of λ_R, to the temperature basis of 21·005° C. applying to the metre determinations is shown in full detail and needs no further explanation.

Only one comparison in each series of eight made in the metre and yard determinations employed the complete group of five radiations. The remaining comparisons were carried out on the same lines but using only the standard radiation λ_R and one auxiliary radiation λ_G, for it was only necessary to check the integral part of the order of interference in λ_R in these comparisons after having once established this order without ambiguity by the observations in the group of five radiations. In such manner the time occupied in taking observations was reduced in the shortened comparisons from about 110 minutes to about 35 minutes.

APPENDIX III.

Comparisons of the Composite Gauges with the Fundamental Units of Length.

By J. E. Sears, *Jnr.*, *C.B.E.*, *M.A.*, *M.I.Mech.E.*, and W. H. Johnson, *B.Sc.*

Advantage was taken of the presence at the Laboratory, for the purpose of the statutory decennial comparisons of standards, of the Imperial Standard Yard, and of the British National Copy of the International Prototype Metre, to obtain comparisons, as direct as possible, between the composite yard and metre gauges and the fundamental standards of the two systems of measurement. The following is a brief account of this part of the work.

(*a*) *The Composite Metre Gauge.*—The composite metre gauge consists of an auxiliary end-bar, whose length is one metre minus half an inch, and two end-blocks, each half an inch between the parallel faces. The end-faces of the bar are finished with high precision so that they are flat and square to the axis of the bar, and are therefore parallel to one another. Both faces are flat to within 0·04μ and are parallel to one another within 0·25μ. The blocks are similarly finished and both faces are flat and parallel within 0·04μ. It is well known that highly polished flat steel surfaces will adhere to each other by "wringing," with an extremely thin separating film between them.* The auxiliary bar plus either of the blocks wrung centrally on to one of its ends constitutes an end-standard nominally one metre long, while with both blocks wrung on it on opposite ends it serves as a line-standard nominally one metre long, as explained by the help of fig. 3.

---

* Rolt and Barrell, 'Proc. Roy. Soc.,' A, vol. 116, p. 401 (1927).

Each block is wrung on so that one of the faces which are perpendicular to the end-faces is in the plane containing the axis of the auxiliary bar. A fine line is scribed on this face, fig. 3, very nearly half-way between the end-faces and parallel to them. The distance between the lines is approximately one metre, the actual distance being determined by comparison with a metre line-standard. Keeping them the same way up, and using only those portions of the end-faces of the bar indicated in the figure, the end-blocks may still be wrung on in four different arrangements, and if M_1, M_2, M_3, and M_4 are the measured lengths between the graduations corresponding to these several conditions, and a, b, p, and q are the approximate half-lengths of the block, t the thickness of each wringing film, and L the length of the auxiliary bar, then the following four equations can be written down :—

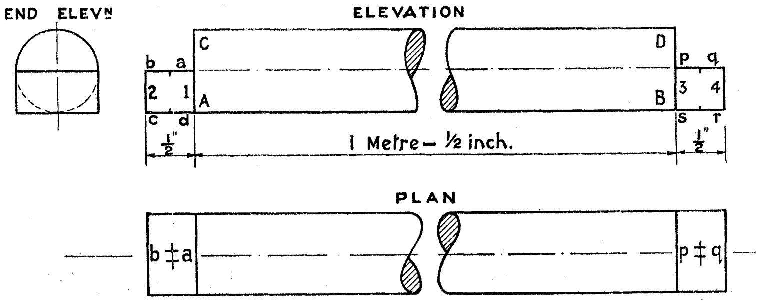

FIG. 3—Diagram of the Composite Metre Gauge.

$$L + (a + p) + 2t = M_1$$

$$L + (b + p) + 2t = M_2$$

$$L + (b + q) + 2t = M_3$$

$$L + (a + q) + 2t = M_4.$$

Adding these and dividing by four, we get

$$L + \tfrac{1}{2}(a + b + p + q) + 2t = \tfrac{1}{4}(M_1 + M_2 + M_3 + M_4).$$

To eliminate the effect of any slight error of parallelism of the end-blocks, the under sides of the blocks were also given graduation lines as indicated in fig. 3, the various portions of these surfaces being designated by the letters c, d, r, and s as shown. Inverting the end-blocks we thus get in a similar manner :—

$$L + \tfrac{1}{2}(c + d + r + s) + 2t = \tfrac{1}{4}(M_5 + M_6 + M_7 + M_8),$$

and hence finally, adding and dividing by 2 :—

$$L + \tfrac{1}{2}(\text{sum of blocks}) + 2t = \frac{1}{8} \cdot \Sigma M.$$

The determination of the effective lengths of the composite standards as expressed by the above formula has been carried out in the Laboratory one-metre comparator by direct comparison with the British National Copy of the Metre, P.I. 16, and with both the metre intervals on the Laboratory metre standard No. 184.* Four complete determinations have been made against each standard, each determination involving sixteen sets of observations, or forty-eight sets in all.

In setting up the standards for observation conditions were varied as much as possible. Half the sets were made with the composite standard on the front girder and half with it on the back girder of the comparator; the relative orientation of the two bars under comparison was varied by turning them end to end; and a further variation made by rotating the auxiliary bar through 180° about its own axis and repeating the observations with the blocks wrung to the opposite halves of its end-faces. Also the microscopes were interchanged in position on the comparator when half the sets had been completed.

The scheme of comparisons is set out in Table XXI, which should be used in conjunction with the symbols shown in fig. 3. The small letters associated with the blocks denote the portions of the blocks that were used in the comparisons while the capital letters associated with the auxiliary bar indicate the parts of its end-faces to which the blocks were wrung.

TABLE XXI.—Scheme of Comparisons for the Composite Metre Gauge.

Position of microscopes X and Y.		Composite metre on front girder.		Composite metre on back girder.	
Left.	Right.	Left.	Right.	Left.	Right.
X	Y	*a* A *c* B	*p* B *r* A	*b* A *d* B	*q* B *s* A
		q A *s* B	*a* B *c* A	*p* A *r* B	*b* B *d* A
Y	X	*b* C *d* D	*s* D *q* C	*a* C *c* D	*r* D *p* C
		r C *p* D	*b* D *d* C	*s* C *q* D	*a* D *c* C

Comparisons with the three standards were made consecutively for each wringing; that is, sixteen wringings only were made.

The observations throughout were made at temperatures close to 20° C., the mean temperature of observation being about 20·13° C., and the extreme range 19·61° C. to 20·67° C.

---

* For description of this bar, see SEARS, JOHNSON, and JOLLY (*loc. cit.*).

Comparisons of P.I. 16 and the two lengths of nickel 184 (distinguished below as 184_1 and 184_2) had previously been made in connection with the intercomparison of a number of metre bars, and if these results be taken into account, one obtains a closed set of comparisons, from which by the method of least squares the value of the length of the composite metre, based on the most recent value of P.I. 16, can be calculated.

The length of P.I. 16 is given by the Bureau International* as :—

$$\text{P.I. 16 at } 0^\circ \text{ C.} = 0\cdot999{,}999{,}34 \text{ M.}$$

This value is based on a re-comparison made in 1922, allowance being made for the latest accepted equations (October, 1933) for the working standards of the Bureau, against which P.I. 16 was compared. A subsequent comparison made early in 1933 led to a somewhat lower result, namely :—

$$\text{P.I. 16 at } 0^\circ \text{ C.} = 0\cdot999{,}999{,}21 \text{ M,}$$

but this comparison is not regarded by the Bureau as entirely satisfactory and therefore has not been taken into account in the present work.

Making use of the former value of P.I. 16, the mean length of the composite metre gauge, ascertained from the whole of the comparisons above described was found to be :—

$$\text{M}' \text{ at } 20^\circ \text{ C.} = 1\cdot000{,}042{,}78 \text{ M.}$$

The results of the various comparisons with M′ are set out in Table XXII.

TABLE XXII.—Results of the Comparisons with M′.

Comparison.	Parts of blocks used.	Observed result reduced to 20° C. (μ).	Mean. (μ).	Difference (obs.-mean). (μ).
P.I. 16 — M′	*a b p q*	+129·74	+129·67	+0·07
	c d s r	+129·81		+0·14
	a b s r	+129·64		—0·03
	c d p q	+129·48		—0·19
184_1 — M′	*a b p q*	+ 90·42	+ 89·66	+0·76
	c d s r	+ 90·16		+0·50
	a b s r	+ 89·06		—0·60
	c d p q	+ 88·99		—0·67
184_2 — M′	*a b p q*	+ 88·53	+ 88·04	+0·49
	c d s r	+ 88·35		+0·31
	a b s r	+ 87·61		—0·43
	c d p q	+ 87·66		—0·38

The differences recorded in this table are greater than would normally have been expected, but exhaustive examination of the individual measurements has failed to

---

* ‘C. R. 8ième Conf. Gén. de Pds. et Mes.,’ 1933, in the press.

reveal any explanation of the comparatively wide range of deviations shown. On the other hand, the mean observed results for all the comparisons involved in the determination of M′ are highly self-consistent, as is shown in Table XXIII.

TABLE XXIII.—Results of all Comparisons Involved in the Determination of M′.

Comparison.	Observed result reduced to 20° C. (μ).	Calculated result. (μ).	Residual (obs.-calc.). (μ).
P.I. 16 — 184_1	+ 40·02	+ 40·06	—0·04
P.I. 16 — 184_2	+ 41·76	+ 41·69	+0·07
P.I. 16 — M′	+129·67	+129·70	—0·03
184_1 — 184_2	+ 1·58*	+ 1·63	—0·05
184_1 — M′	+ 89·66	+ 89·64	+0·02
184_2 — M′	+ 88·04	+ 88·01	+0·03

* Calibration value, see SEARS, JOHNSON, and JOLLY (*loc. cit.*).

(*b*) *The Composite Yard Gauge.*—The composite yard gauge consists of an auxiliary end-bar $35\frac{1}{2}$ inches long and two end-blocks each half an inch between the parallel faces. The manner in which it can be used either as an end-standard or as a line-standard is precisely the same as for the composite metre and the determination of its effective length when used as a line-standard is carried out on the same lines. Actually the same end-blocks were used. Both faces of the auxiliary bar are flat to within $0{\cdot}04\mu$, and are parallel to one another within $0{\cdot}15\mu$.

The verification of the effective length of the gauge was carried out in the Laboratory one-metre comparator by direct comparison with the Imperial Standard Yard and with the Yard interval 2/38 inches of the Laboratory nickel standard No. 184. Four complete determinations were made against each standard, involving sixteen sets of observations or thirty-two sets in all.

The scheme of comparisons is set out in Table XXIV.

Comparisons with the two standards were made consecutively for each wringing: that is, sixteen wringings only were made.

The observations throughout were made at temperatures close to 16·67° C. (62° Fahr.), the mean temperature of observation being about 16·85° C., and the extreme range 16·44° C. to 17·15° C.

The length of Nickel 184 had previously been determined against the Imperial Standard Yard in a closed set of yard bars, and this result, combined in the usual way with the observed results, gives an adjusted value of the composite yard, based on the length of the Imperial Standard.

The length of the composite yard thus determined was found to be :—

$$36 \text{ inches} - 0{\cdot}81\mu \text{ at } 62^\circ \text{ F.}$$

$$\text{or } Y' \text{ at } 62^\circ \text{ F. } (16{\cdot}666_7{}^\circ \text{ C.}) = 0{\cdot}999{,}999{,}11 \text{ Y.}$$

TABLE XXIV.—Scheme of Comparisons for the Composite Yard Gauge.

Position of microscopes X and Y.		Composite yard on front girder.		Composite yard on back girder.	
Left.	Right.	Left.	Right.	Left.	Right.
X	Y	*b* A *d* B	*q* B *s* A	*a* A *c* B	*p* B *r* A
		p A *r* B	*b* B *d* A	*q* A *s* B	*a* B *c* A
Y	X	*a* C *c* D	*r* D *p* C	*b* C *d* D	*s* D *q* C
		s C *q* D	*a* D *c* C	*r* C *p* D	*b* D *d* C

The results of the various comparisons with Y′ are shown in Table XXV.

It should be explained that the determination of the difference in length between Y and Y′ was made in terms of divisions of the scales of the eye-piece micrometers, fitted to the comparator microscopes, which are arranged to read in terms of the metric system.

TABLE XXV.—Results of the Comparisons with Y′.

Comparison.	Parts of blocks used.	Observed result at 16·67° C. (μ).	Mean. (μ).	Difference (obs.-mean). (μ).
Imp. Std. — Y′	*a b p q*	+ 1·34	+ 0·81	+0·53
	c d s r	+ 1·05		+0·24
	a b s r	+ 0·46		—0·35
	c d p q	+ 0·38		—0·43
184 — Y′	*a b p q*	—32·89	—33·01	+0·12
	c d s r	—32·96		+0·05
	a b s r	—33·04		—0·03
	c d p q	—33·15		—0·14

The differences in the first section of Table XXV are again somewhat larger than expected, but, as in the case of the metric comparisons, the mean results of all comparisons involved in the determination of Y′, shown in Table XXVI, are seen to be highly self-consistent.

TABLE XXVI.—Results of all Comparisons Involved in the Determination of Y′.

Comparison.	Observed result at 62° F. (μ).	Calculated result. (μ).	Residual (obs.-calc.). (μ).
Imp. Std. — 184	+33·80	+33·81	—0·01
Imp. Std. — Y′	+ 0·81	+ 0·81	0·00
184 — Y′	—33·01	—33·00	—0·01

(*c*) *Expansion Formulæ.*—The following are the expansion formulæ adopted for the five bars involved in the various metrological comparisons described above, together with the basis of determination of each.

P.I. 16.

$$L_t = L_0 \,[1 + (8{\cdot}6210\, t + 0{\cdot}00180\, t^2)\, 10^{-6}]$$

Basis.—Determined by the Bureau International des Poids et Mesures. It is the common formula now accepted for all the national copies of the Prototype Metre.

Imperial Standard Yard.

$$L_t = L_0 \,[1 + 17{\cdot}748 \times 10^{-6} t]$$

Basis.—Determination by A. R. CLARKE, "Comparisons of Standards of Length," H.M.S.O. (1866), mean of two values pp. 212 and 216.

Ni 184.

$$L_t = L_0 \,[1 + (12{\cdot}450\, t + 0{\cdot}00647\, t^2)\, 10^{-6}]$$

Basis.—Determined against N.P.L. Nickel 16, which was determined by the Bureau International des Poids et Mesures in 1913 against the Bureau standard P.I. 13_{74}. Adjustments to the value have since been made to correspond with adjustments to P.I. 13_{74}.

Composite Metre Gauge.

$$L_t = L_0 \,[1 + (10{\cdot}540\, t + 0{\cdot}00677\, t^2)\, 10^{-6}]$$

Basis.—Determined against N.P.L. Invar 27, which was determined by the Bureau International des Poids et Mesures in 1902. The value of Invar 27 has since been adjusted for the same reason as that of Nickel 16.

Composite Yard Gauge.

$$L_t = L_0 \,[1 + (10{\cdot}492\, t + 0{\cdot}00580\, t^2)\, 10^{-6}]$$

Basis.—As for the composite metre gauge.